

EE 562

Homework 10

Due Wednesday, April 19, 2017 at 6:40 p.m.

Work all 5 problems.

Problem 1. Consider the mean square differential equation

$$\frac{dY(t)}{dt} + 2Y(t) = X(t)$$

for $t > 0$ subject to the initial condition $Y(0) = 0$. The input is

$$X(t) = 5 \cos 2t + W(t)$$

where $W(t)$ is a white Gaussian noise process with mean zero and covariance function $K_W(\tau) = \sigma^2 \delta(\tau)$. Find the covariance $K_Y(t_1, t_2)$ for $t_1, t_2 > 0$.

Problem 2. Suppose X is a Poisson random variable with parameter λ . Then

$$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k = 0, 1, 2, \dots$$

Show that $E[X] = \lambda$ and $Var[X] = \lambda$.

Problem 3. Let $N(t)$ be a Poisson process with parameter λt . Find

$$E [(N(t) - N(s))^2]$$

for $t > s$.

Problem 4. Give an example of a random process that is WSS but not ergodic in mean.

Problem 5. Let $X(t)$ be a WSS random process. Show that

$$\frac{\partial^2}{\partial t_1 \partial t_2} R_x(t_1, t_2) = -\frac{d^2}{d\tau^2} R_x(\tau).$$