EE 562

Homework 9

Due Wednesday, April 12, 2017 at 6:40 p.m.

Work all 5 problems.

Problem 1. The wide sense stationary process $X(t) = X(u,t)$ has correlation function

$$R_X(\tau) = \frac{1}{2}e^{-|\tau|}.$$

Let

$$\mu_T = \frac{1}{2T}\int_{-T}^{T} X(t)dt$$

as given in class. Compute $Var(\mu_T)$.

Problem 2. Consider the mean square differential equation

$$\frac{dY(t)}{dt} + 2Y(t) = X(t)$$

for $t > 0$ subject to the initial condition $Y(0) = 0$. The input is

$$X(t) = 5\cos 2t + W(t)$$

where $W(t)$ is a white Gaussian noise process with mean zero and covariance function $K_W(\tau) = \sigma^2\delta(\tau)$. Find $\mu_Y(t)$ for $t > 0$.

Problem 3. Let $X(t) = X(u,t)$ be a WSS random process with correlation function

$$R_X(\tau) = e^{-\tau^2}.$$

a. Show that the mean-square derivative $\dfrac{dX(t)}{dt}$ exists.

b. Find the correlation function of the process $\dfrac{dX(t)}{dt}$.

Problem 4. A wide sense stationary random process $X(t)$ has a covariance function

$$K_X(\tau) = 2\exp(-|\tau|).$$

Is $X(t)$ is ergodic in mean? Justify your answer.

Problem 5. Let $X(t)$ be a Wiener process. Recall $X(t)$ could be obtained as a limiting random walk where each step (up or down) was taken with probability $p = 1/2$. We derived

$$E[X(t)] = 0, \quad K_X(t_1, t_2) = \alpha \min(t_1, t_2)$$

where α was related to the step size and time increment. Let

$$Y(t) = \int_0^t X(v)dv.$$

a. Find the mean of $Y(t)$.

b. Find the variance of $Y(t)$.