EE 562

Homework 7

Due Wednesday, March 29, 2017 at 6:40 p.m.

Work all 5 problems.

Problem 1. Let $X_1, X_2, \ldots, X_n$ denote an i.i.d. sequence of random variables each normally distributed with mean μ and variance σ^2 (both unknown). Derive the MLEs for μ and σ^2.

Problem 2. Let $X_1, X_2, \ldots, X_n$ denote an i.i.d. sequence of random variables each uniformly distributed in $[\theta,\ \theta+1]$. Find an MLE for θ.

Problem 3. Consider a random sequence $x(n)$ with zero mean and covariance

$$K_X(m,n) = a^{|m-n|}.$$

A second sequence $y(n)$ is generated as

$$y(n) = x(n) - x(n-1).$$

a. Compute the cross covariance of x and y and the corresponding cross spectral density.

b. Compute the covariance of y and its power spectral density.

Problem 4. A zero mean sequence of i.i.d. random variables $x(n)$ form the input to a causal linear system defined by the linear difference equation

$$y(n) = \alpha y(n-1) + x(n), \quad |\alpha| < 1.$$

Find the impulse response $h(n)$ of a second system such that the sequence

$$z(n) = \sum_{i=0}^{k} h(i) y(n-i)$$

has a constant power spectral density, i.e.,

$$S_Z(f) = \sigma_z^2, \quad f \in \left[-\frac{1}{2},\ \frac{1}{2}\right).$$

Problem 5. Suppose that you can observe a signal which is the sum of two mutually orthogonal, WSS, zero-mean random sequences which is passed through a known linear system, i.e.,

$$y(n) = \sum_{k=0}^{N} h(k)[x_1(n-k) + x_2(n-k)].$$

Assume their covariances are known.

a. Construct a Wiener filter to optimally recover the sequence $x_1(n)$ from $y(n)$.

b. Suppose that the signal is corrupted with additive noise, i.e.,

$$z(n) = y(n) + w(n)$$

where $w(n)$ is WSS and uncorrelated with the sequences $x_1(n)$ and $x_2(n)$. Find an optimal Wiener filter to recover $x_1(n)$ from $z(n)$.