EE 562

Homework 5

Due Wednesday, February 22, 2017 at 6:40 p.m.

Work all 5 problems.

Problem 1. Let $R_i,\ i = 1, \ldots, 32$ be 32 independent random variables resulting from the envelope detection of a signal plus noise process. Assume that each R_i resulted from the envelope detection of a complex signal plus noise component.

Assuming integration detection with $N = 32$ use Albersheim's equation to plot probability of detection (P_d) vs. SNR (dB) for a probability of false alarm (P_{fa}) of 10^{-6}.

Problem 2. Same setup as Problem 1. We know that the density of each R_i when no signal is present is

$$f_{R_i}(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, \quad r \geq 0$$

where σ^2 is the total noise power.

With M of N detection a threshold is found for each of the 32 samples as

$$T_0 = \sqrt{-2\sigma^2 \ln(P_{fa,s})}$$

where $P_{fa,s}$ is the probability of false alarm on a sample basis that yields an overall P_{fa}.

a. Using

$$P_{fa} = \sum_{K=M}^{N} \binom{N}{K} P_{fa,s}^K (1 - P_{fa,s})^{N-K}$$

find $P_{fa,s}$ that yields an overall $P_{fa} = 10^{-6}$ where $M = 16$ and $N = 32$.

b. Using

$$P_d = \sum_{K=M}^{N} \binom{N}{K} P_{d,s}^K (1 - P_{d,s})^{N-K}$$

find $P_{d,s}$, the probability of detection on a sample basis, that yields an overall $P_d = 0.9$ where $M = 16$ and $N = 32$.

Problem 3. Let $\{x_n\}$ be an orthonormal set in a pre-Hilbert (or inner product space) H. Show for any x in H

$$\sum_n |< x, x_n >|^2 \le ||x||^2.$$

Recall that $\{x_n\}$ an orthonormal set means $< x_m, x_n >= 0$ if $m \neq n$ and $< x_n, x_n >= 1$.

Problem 4. Suppose X and Y are correlated Gaussian random variables each with mean zero and unity variance with correlation coefficient ρ.

a. Write down the joint density of (X, Y).

b. Write down the 2×2 correlation matrix $\mathbf{R_{XY}}$.

Problem 5. This is a continuation of Problem 4. Suppose we do not know the underlying distribution for (X, Y) but have 2-dim samples (X_k, Y_k), $k = 1, 2, \ldots n$ from the distribution. We wish to use these samples to estimate the correlation matrix. For this problem you may assume that $\rho = 0.5$.

To generate X and Y in Matlab we may use the following technique. First, use a Matlab function call to get a standard normal random deviate. Call this X. Now get another standard normal random deviate. Call this Z. Now let

$$Y = \rho X + \sqrt{1 - \rho^2} Z.$$

a. Verify (analytically) that Y is standard normal and $E[XY] = \rho$.

b. Using Matlab estimate the correlation matrix using $n = 50$. Then subtract the actual correlation matrix found in Problem 4 (with $\rho = 0.5$) from your estimated correlation matrix. Call this matrix result $\mathbf{E_R}$.

c. Using $n = 50$ samples in part (b) your estimate of the correlation matrix was a single result. Now repeat this process $N = 1000$ times and compute the mean and variance of each entry of your 1000 $\mathbf{E_R}$ matrices.

d. Repeat part (c) but now use $n = 500$ (N is still 1000).

e. Comment on how your mean and variance results compare when using $n = 50$ and $n = 500$.