EE 562

Homework 4

Due Wednesday, February 15, 2017 at 6:40 p.m.

Work all 5 problems.

Problem 1. Suppose the random sequence Z_n converges to Z in distribution where Z is some constant z_0 and each Z_n is a continuous random variable. In this case show that $Z_n \to Z$ in distribution implies $Z_n \to Z$ in probability.

Problem 2. Let $X_n,\ n \geq 1$, denote a sequence of independent and identically distributed zero-mean unit-variance Gaussian random variables.

Define for $n \geq 1$

$$Y_n = (1 - \alpha)X_n + \alpha Y_{n-1}$$

where $Y_0 = 0$ and $0 < \alpha < 1$. Since each Y_n is a linear combination of Gaussian random variables we know each Y_n is a Gaussian random variable. Find the limiting mean and variance of this Gaussian random sequence, i.e., compute the mean and variance of Y_n as $n \to \infty$.

Problem 3. Design a correlation detector for deciding whether H_0 or H_1 is true where

$$H_i:\ \mathbf{X}(u) = \mathbf{S}_i + \mathbf{N}(u), \quad i = 0, 1$$

and

$$\mathbf{S}_i = (-1)^i \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$E[\mathbf{N}(u)] = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad \mathbf{K_N} = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}.$$

Indicate the decision regions on a graph.

Problem 4. We are given an observation of X and we must decide between two hypotheses:

$$H_0 : \quad X = N$$
$$H_1 : \quad X = S_1 + N$$

where, N is a continuous random variable with density

$$f_N(n) = \begin{cases} 2e^{-2n}, & n > 0, \\ 0, & \text{elsewhere} \end{cases}$$

and $S_1 = 2.1$.

a. Find a threshold T so that the probability of type I error is 0.01.

b. Using the T you found find the probability of type II error.

c. What is the power of this test?

Problem 5. We are given an observation of X and we must decide between two hypotheses:

$$H_0 : \quad X = N$$
$$H_1 : \quad X = S_1 + N$$

where, N is a continuous random variable with density

$$f_N(n) = \begin{cases} n+1, & -1 \le n \le 0, \\ 1-n, & 0 < n \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a. Find a threshold T (to 3 decimal places) so that the probability of type I error is 0.001.

b. Find the value of S_1 so that the probability of type II error is 0.01.