EE 562

Homework 3

Due Wednesday, February 8, 2017 at 6:40 p.m.

Work all 5 problems.

Problem 1. Suppose the continuous random variable U is uniform on [0,1]. Determine for each of the following if the sequence converges and, if so, in what sense and to what limiting random variable.

a. $X_n(u) = u^n$.

b. $Z_n(u) = cos^n(2\pi u)$.

Problem 2. Let X_n be a sequence of i.i.d. equiprobable Bernoulli (0,1) random variables and let

$$Y_n = 2^n X_1 X_2 \cdots X_n.$$

a. Does Y_n converge almost surely and, if so, to what limit?

b. Does Y_n converge in the mean square sense and, if so, to what limit?

Problem 3. Let X_n be a sequence of i.i.d. random variables with mean μ and variance $\sigma^2 < \infty$. Let

$$Y_n = \frac{1}{n} \sum_{k=1}^{n} X_k.$$

Show that Y_k converges in the mean square sense.

Problem 4. Let X_n and Y_n denote two (possibly dependent) sequences of random variables that converge in the mean square sense to X and Y, respectively. Does the sequence $Z_n = X_n + Y_n$ converge in the mean square sense and, if so, to what limit?

Problem 5. Suppose U is uniform on [0,1]. Define

$$X_n(u) = \sum_{k=1}^{n} \frac{1}{u+k}.$$

Does $X_n(u)$ converge and, if so, in what sense and to what limit?