EE 562

Homework 2

Due Wednesday, February 1, 2017 at 6:40 p.m.

Work all 5 problems.

Problem 1. Are there any real values of β that would permit the following matrix to be a correlation matrix? If so, find them. If not, show why not.

$$A = \begin{bmatrix} 3-\beta & 2 \\ 2 & 1+\beta \end{bmatrix}.$$

Problem 2. Let $\mathbf{W}(u)$ be a white random vector with

$$\mu_W = (0 \quad 0 \quad 0)^t, \quad \mathbf{K}_W = \mathbf{I}.$$

Let

$$\mathbf{X}(u) = \mathbf{H}\mathbf{W}(u) + \mathbf{c}.$$

Find $\mathbf{c}$ and a causal matrix $\mathbf{H}$ using the direct method that produces

$$\mu_X = [1 \;\; 2 \;\; 3]^T, \quad \mathbf{K}_X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 6 \end{bmatrix}.$$

Problem 3. Stark and Woods 5.29.

Problem 4. Stark and Woods 5.34.

Problem 5. Let $\mathbf{X}(u)$ be an n-dimensional random vector with covariance matrix $\mathbf{K}_X$ and correlation matrix $\mathbf{R}_X$. Let $(\lambda_i, \; \mathbf{e}_i), \; i = 1, 2, \ldots, n$ denote the eigenvalue and eigenvector pairs of the covariance matrix $\mathbf{K}_X$, with the eigenvectors chosen to form an orthonormal set.

a. If $\mathbf{K}_X$ is non-singular, can $\mathbf{R}_X$ be singular? Why or why not?

b. Show $\mathbf{K}_X$ can be written in the form

$$\mathbf{K}_X = \sum_{i=1}^{n} \lambda_i \mathbf{e}_i \mathbf{e}_i^\dagger.$$

c. Construct an example in which $\mathbf{R}_X$ is non-singular but $\mathbf{K}_X$ is singular.

d. Construct an example in which $\mathbf{K}_X$ and $\mathbf{R}_X$ are both singular but $\mu_X \neq 0$.

e. Verify that

$$\mathbf{R}_X^{-1} = \mathbf{K}_X^{-1} - \frac{\mathbf{K}_X^{-1}\mu_X\mu_X^{\dagger}\mathbf{K}_X^{-1}}{1 + \mu_X^{\dagger}\mathbf{K}_X^{-1}\mu_X}.$$