EE 562

Homework 1

Due Wednesday, January 25, 2017 at 6:40 p.m.

Work all 5 problems.

Problem 1. Suppose the random variable X has density

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

a. Compute $P(X < 1)$.

b. Let $Y = X^2 + 1$. Find $E(Y)$.

Problem 2.

a. Suppose X is exponentially distributed with parameter λ, i.e.,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Derive the moment generating function for X.

b. Suppose the random variable Y has moment generating function

$$M_Y(s) = (1 - 2s)^{-1}, \quad s < \frac{1}{2}.$$

Find the mean and variance of Y.

Problem 3. Compute the eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}.$$

Problem 4. Suppose $Y(u) = A^T X(u) + b$ where

$$A^T = [2 \quad -1 \quad 2], \quad \mu_X^T = [5 \quad -5 \quad 6], \quad b = 5$$

and let

$$K_X = \begin{bmatrix} 5 & 2 & -1 \\ 2 & 5 & 0 \\ -1 & 0 & 4 \end{bmatrix}.$$

a. Compute the mean of $Y(u)$.

b. Compute the variance of $Y(u)$.

Problem 5. Suppose $\mathbf{Y}(u)$ is formed by a linear transformation of $\mathbf{X}(u)$. Here $\mathbf{X}(u) \in \mathbf{R}^n$ and $\mathbf{Y}(u) \in \mathbf{R}^m$.

$$\mathbf{Y}(u) = \mathbf{HX}(u)$$

where

$$H = \begin{bmatrix} h_{11} & \dots & h_{1n} \\ \vdots & \dots & \vdots \\ h_{m1} & \dots & h_{mn} \end{bmatrix}.$$

Show $\mathbf{K_Y} = \mathbf{HK_XH}^\dagger$.