

7.0 Spectral Whitening

Consider the following two systems:

$$\mathbf{X} = \mathbf{H}\mathbf{W}$$

$$\mathbf{Y} = \mathbf{A}\mathbf{X}$$

Given zero-mean \mathbf{X} with covariance $\mathbf{K}_\mathbf{X}$ we wish to find \mathbf{A} so that \mathbf{Y} has covariance \mathbf{I} . We know

$$\mathbf{H} = \mathbf{E}_\mathbf{X}\Lambda_\mathbf{X}^{1/2}\mathbf{U}.$$

If \mathbf{H}^{-1} exists we set $\mathbf{A} = \mathbf{H}^{-1} = (\mathbf{E}_\mathbf{X}\Lambda_\mathbf{X}^{1/2}\mathbf{U})^{-1}$. Then

$$\begin{aligned}\mathbf{K}_\mathbf{Y} &= \mathbf{A}\mathbf{K}_\mathbf{X}\mathbf{A}^\dagger \\ &= \mathbf{H}^{-1}\mathbf{H}\mathbf{H}^\dagger (\mathbf{H}^{-1})^\dagger = \mathbf{I}.\end{aligned}$$

Now $\mathbf{A} = \mathbf{U}^{-1}\Lambda_\mathbf{X}^{-1/2}\mathbf{E}_\mathbf{X}^{-1}$ or

$$\mathbf{A} = \mathbf{U}^\dagger\Lambda_\mathbf{X}^{-1/2}\mathbf{E}_\mathbf{X}^\dagger.$$