

## 7.0 Spectral Whitening

Consider the following two systems:

$$\mathbf{X} = \mathbf{H}\mathbf{W}$$

$$\mathbf{Y} = \mathbf{A}\mathbf{X}$$

Given zero-mean  $\mathbf{X}$  with covariance  $\mathbf{K}_\mathbf{X}$  we wish to find  $\mathbf{A}$  so that  $\mathbf{Y}$  has covariance  $\mathbf{I}$ . We know

$$\mathbf{H} = \mathbf{E}_\mathbf{X}\Lambda_\mathbf{X}^{1/2}\mathbf{U}.$$

If  $\mathbf{H}^{-1}$  exists we set  $\mathbf{A} = \mathbf{H}^{-1} = (\mathbf{E}_\mathbf{X}\Lambda_\mathbf{X}^{1/2}\mathbf{U})^{-1}$ . Then

$$\begin{aligned}\mathbf{K}_\mathbf{Y} &= \mathbf{A}\mathbf{K}_\mathbf{X}\mathbf{A}^\dagger \\ &= \mathbf{H}^{-1}\mathbf{H}\mathbf{H}^\dagger (\mathbf{H}^{-1})^\dagger = \mathbf{I}.\end{aligned}$$

Now  $\mathbf{A} = \mathbf{U}^{-1}\Lambda_\mathbf{X}^{-1/2}\mathbf{E}_\mathbf{X}^{-1}$  or

$$\mathbf{A} = \mathbf{U}^\dagger\Lambda_\mathbf{X}^{-1/2}\mathbf{E}_\mathbf{X}^\dagger.$$