EE 562a

Homework 9

(not to be handed in)

Work the following 5 problems.

Problem 1. Consider the mean square differential equation

$$\frac{dY(t)}{dt} + 2Y(t) = X(t)$$

for $t > 0$ subject to the initial condition $Y(0) = 0$. The input is

$$X(t) = e^{-t} + W(t)$$

where $W(t)$ is a white Gaussian noise process with mean zero and covariance function $K_W(\tau) = \sigma^2\delta(\tau)$.

a. Find $\mu_Y(t)$ for $t > 0$.

b. Find the covariance $K_Y(t_1, t_2)$ for $t_1, t_2 > 0$.

Problem 2. Let $N(t)$ be a Poisson process with parameter λ. Find

$$E\left[(N(t) - N(s))^2\right]$$

for $t > s$.

Problem 3. Let U be a random variable uniformly distributed on $(0, 1)$. Show that $1 - U$ is also uniformly distributed on $(0, 1)$.

Problem 4. Show how to use the inverse transform method to generate a random variable X having density function

$$f(x) = \frac{e^x}{e - 1},\ 0 \le x \le 1.$$

Problem 5. Let

$$\theta = \int_0^1 e^{x^2} dx.$$

a. Show how you can use two independent uniform random variables to estimte θ.

b. Show how you can use antithetic variables to estimte θ.

c. Show that the variance of the estimator in part (b) is less than the variance of the estimator in part (a).