# EE 562a

## Homework 8

Due Wednesday July 29, 2009

**Work the following 7 problems**.

**Problem 1.** A zero mean sequence of i.i.d. random variables $x(n)$ form the input to a causal linear system defined by the linear difference equation

$$y(n) = \alpha y(n-1) + x(n), \quad |\alpha| < 1.$$

Find the impulse response $h(n)$ of a second system such that the sequence

$$z(n) = \sum_{i=0}^{k} h(i) y(n-i)$$

has a constant power spectral density, i.e.,

$$S_Z(f) = \sigma_z^2, \quad f \in \left[-\frac{1}{2}, \frac{1}{2}\right).$$

**Problem 2.** A random process $X(t)$ is given by

$$X(t) = e^{-Yt} u(t)$$

where $Y$ is a uniform random variable in the interval [0, 1] and $u(t)$ is the unit step function. Find $P(X(t) \leq 0.25)$.

**Problem 3.** Let $X(t)$ be a stationary random process with mean $\mu_X$ and covariance function

$$K_X(\tau) = \frac{\sigma_X^2}{1+\tau^4}, \quad \infty < \tau < \infty.$$

a. Show that the mean square derivative exists for all $t$.

b. Find $\mu_{X'}(t)$ and $K_{X'}(\tau)$.

**Problem 4.** A stationary random process $X(t)$ has an autocorrelation function

$$R_X(\tau) = 10 \exp(-|\tau|).$$

Show that $X(t)$ is ergodic in mean.

**Problem 5.** Give an example of a random process that is WSS but not ergodic in mean.

**Problem 6.** Let $X(t)$ be a WSS random process. Show that

$$\frac{\partial^2}{\partial t_1 \partial t_2} R_x(t_1, t_2) = -\frac{d^2}{d\tau^2} R_x(\tau).$$

**Problem 7.** Using the notation and definitions in the K-L theorem, let $X(t)$ be a random process and

$$\hat{X}(t) = \sum_{n=1}^{\infty} X_n \phi_n(t)$$

where

$$X_n = \int_{-T/2}^{T/2} X(t) \phi_n^*(t) dt.$$

Show (as stated in class) that

$$E\left[|\hat{X}(t) - X(t)|^2\right] = 0.$$