EE 562a

Homework 4

Due Wednesday June 17, 2009

Work the following 5 problems.

Problem 1. Design a correlation detector for deciding whether H_0 or H_1 is true where

$$H_i: \ \mathbf{X}(u) = \mathbf{S}_i + \mathbf{N}(u), \quad i = 0, 1$$

and

$$\mathbf{S}_0 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \mathbf{S}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$E[\mathbf{N}(u)] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{K}_{\mathbf{N}} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}.$$

Indicate the decision regions on a graph.

Problem 2. We are given an observation of X and we must decide between two hypotheses:

$$H_0: \quad X = N$$

$$H_1: \quad X = S + N$$

where N has density

$$f_N(x) = \frac{3}{2} e^{-3|x|}, \quad x \in (-\infty, \infty)$$

and $S = 4.5$.

a. Find a threshold T so that the probability of type I error is 10^{-5}. Note: You can assume S will always be greater than 0 when it appears so the T you find should be greater than 0, i.e., the negative tail-end of the noise density does not cause a false alarm.

b. Using the T you found find the probability of type II error.

c. What is the power of this test?

Problem 3. Let $R_i,\ i = 1, \ldots, 32$ be 32 independent random variables resulting from the envelope detection of a signal plus noise process. Assume that each R_i resulted from the envelope detection of a complex signal plus noise component.

Assuming integration detection with $N = 32$ use Albersheim's equation to plot probability of detection (P_d) vs. SNR (dB) for a probability of false alarm (P_{fa}) of 10^{-6}.

Problem 4. Same setup as Problem 3. We know that the density of each R_i when no signal is present is

$$f_{R_i}(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, \quad r \geq 0$$

where σ^2 is the total noise power.

With M of N detection a threshold is found for each of the 32 samples as

$$T_0 = \sqrt{-2\sigma^2 \ln(P_{fa,s})}$$

where $P_{fa,s}$ is the probability of false alarm on a sample basis that yields an overall P_{fa}.

a. Using

$$P_{fa} = \sum_{K=M}^{N} \binom{N}{K} P_{fa,s}^K (1 - P_{fa,s})^{N-K}$$

find $P_{fa,s}$ that yields an overall $P_{fa} = 10^{-6}$ where $M = 16$ and $N = 32$.

b. Using

$$P_d = \sum_{K=M}^{N} \binom{N}{K} P_{d,s}^K (1 - P_{d,s})^{N-K}$$

find $P_{d,s}$, the probability of detection on a sample basis, that yields an overall $P_d = 0.95$ where $M = 16$ and $N = 32$.

c. Using

$$P_{d,s} = \int_{T_0}^{\infty} \frac{r}{\sigma^2} e^{-(r^2+s^2)/2\sigma^2} I_0\left(\frac{rs}{\sigma^2}\right) dr$$

find the $SNR = \frac{s^2}{2\sigma^2}$ required to yield the desired $P_{d,s}$ (express your SNR in dB).

d. Compare the SNR required for a $P_d = 0.95$ in this problem to the SNR required in Problem 3 using integration detection.

Problem 5. Same setup as Problems 3 and 4. In the above problems we used $M = N/2$. Using $P_{fa} = 10^{-6}$ and $P_d = 0.95$ plot required SNR (dB) vs. M for $N = 32$. Use $M = 1, 2, \ldots, 32$ in your plot.