EE 562a

Homework 2

Due Wednesday June 3, 2009

Work the following 6 problems.

Problem 1. Given the covariance matrix

$$\mathbf{K} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 12 \\ 3 & 12 & 27 \end{bmatrix}.$$

Factor $\mathbf{K}$ as $\mathbf{K} = \mathbf{H}\mathbf{H}^{\dagger}$.

Problem 2. Let $\mathbf{W}(u)$ be a white random vector with

$$\mu_W = (0 \quad 0 \quad 0)^T, \quad \mathbf{K}_W = \mathbf{I}.$$

Let

$$\mathbf{X}(u) = \mathbf{H}\mathbf{W}(u) + \mathbf{c}.$$

Find $\mathbf{c}$ and a causal matrix $\mathbf{H}$ using the direct method that produces

$$\mu_X = [4 \;\; 1 \;\; 3]^T, \quad \mathbf{K}_X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

Problem 3. Stark and Woods 5.22. In this problem you can use Matlab or any other software tool you wish.

Problem 4. Let $\mathbf{X}(u)$ be a random vector with correlation matrix $\mathbf{R}_{\mathbf{X}}$. Let $\mathbf{e}_1$ and $\mathbf{e}_2$ be eigenvectors corresponding to distinct eigenvalues λ_1 and λ_2, respectively. Assume that

$$||\mathbf{e}_1|| = ||\mathbf{e}_2|| = 1.$$

Let

$$Y_i(u) = \mathbf{e}_i^{\dagger}\mathbf{X}(u), \quad i = 1, 2.$$

a. Compute $E[|Y_1(u)|^2]$.

b. Compute $E[Y_1(u)Y_2(u)^*]$.

Problem 5. Suppose $\mathbf{X}$ is a mean-zero random vector with covariance matrix

$$\mathbf{K_X} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 12 \\ 3 & 12 & 27 \end{bmatrix}.$$

Find a transformation $\mathbf{A}$ such that $\mathbf{Y} = \mathbf{AX}$ has covariance matrix $\mathbf{K_Y} = \mathbf{I}$.

Problem 6. Suppose $Z \sim N(0,1)$ and let $Y = XZ$ where X is ± 1 Bernoulli with $P(X = 1) = P(X = -1) = 1/2$.

a. Show $Y \sim N(0,1)$ by deriving the density function of Y.

b. Show $W = Z + Y$ is not normal.