

EE 503

Lecture Notes Part 2

Christopher Wayne Walker, Ph.D.

2.0 Set Operations and Notation

Definition: A *set* is a collection of elements.

Examples:

- i. $\mathbf{Z} = \{\text{all integers}\}$.
- ii. $\mathbf{R} = \{\text{all real numbers}\}$.
- iii. Let $\mathbf{A} = \{-2, 3, 7\}$. Then \mathbf{A} is a *subset* of \mathbf{Z} . We write $\mathbf{A} \subset \mathbf{Z}$ or $\mathbf{A} \subseteq \mathbf{Z}$. The number 3 is an *element* of \mathbf{A} . We write $3 \in \mathbf{A}$.

Definition: The *empty set* is the set that contains no elements. We write $\{\emptyset\}$ or just \emptyset for the empty set.

Suppose we have a set \mathbf{S} with three elements: $\mathbf{S} = \{a, b, c\}$. What are the possible subsets of \mathbf{S} . They are $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, c\}$, \emptyset .

Note that the set itself and the empty set are always subsets of the given set.

Remark: Given a set \mathbf{S} with n elements, there are 2^n subsets of \mathbf{S} .

Definition: The family of all subsets of a set \mathbf{X} is denoted $P(\mathbf{X})$ and is called the *power set* of \mathbf{X} .

Let \mathbf{S} be any set and let \mathbf{A}, \mathbf{B} be two subsets of \mathbf{S} . Then $\mathbf{A} \cup \mathbf{B}$ denotes the union of \mathbf{A} and \mathbf{B} and $\mathbf{A} \cap \mathbf{B}$ denotes the intersection of \mathbf{A} and \mathbf{B} .

- i. If $\mathbf{D} = \mathbf{A} \cup \mathbf{B}$, then \mathbf{D} is also a subset of \mathbf{S} , i.e., $\mathbf{D} \subset \mathbf{S}$ and $d \in \mathbf{D}$ *iff* (if and only if) $d \in \mathbf{A}$ or $d \in \mathbf{B}$. Some authors write $\mathbf{A} + \mathbf{B}$ for $\mathbf{A} \cup \mathbf{B}$.
- ii. If $\mathbf{E} = \mathbf{A} \cap \mathbf{B}$, then $\mathbf{E} \subset \mathbf{S}$ and $e \in \mathbf{E}$ *iff* $e \in \mathbf{A}$ and $e \in \mathbf{B}$. Some authors write \mathbf{AB} for $\mathbf{A} \cap \mathbf{B}$.
- iii. **Definition:** \mathbf{A}^C or $\overline{\mathbf{A}}$ is called the *complement* of \mathbf{A} and consists of all those elements of \mathbf{S} that are not in \mathbf{A} . More specifically, we say $\overline{\mathbf{A}}$ is the *complement* of \mathbf{A} *in* \mathbf{S} but this is shortened when there is no confusion as to what the encompassing set is.

iv. General Notes:

$$\mathbf{A} \cup \overline{\mathbf{A}} = \mathbf{S}, \quad \mathbf{A} \cap \overline{\mathbf{A}} = \emptyset, \quad \overline{\overline{\mathbf{A}}} = \mathbf{A}, \quad \mathbf{A} \cap \emptyset = \emptyset, \quad \mathbf{A} \cup \emptyset = \mathbf{A},$$

$$\overline{\mathbf{S}} = \emptyset, \quad \overline{\emptyset} = \mathbf{S}, \quad \mathbf{A} \cap \mathbf{S} = \mathbf{A}, \quad \mathbf{A} \cup \mathbf{S} = \mathbf{S}.$$

Venn diagrams are often useful for ascertaining truths and properties of sets and their operations. These will be demonstrated in class.

Demorgan's Rules:

$$\overline{\mathbf{A} \cup \mathbf{B}} = \overline{\mathbf{A}} \cap \overline{\mathbf{B}}, \quad \overline{\mathbf{A} \cap \mathbf{B}} = \overline{\mathbf{A}} \cup \overline{\mathbf{B}}.$$