EE 503

Quiz 13 Solution

Fall 2019, 15 Minutes, 15 Points

Problem 1. (5 points.) Suppose you are going to conduct a political poll for Candidate A and Candidate B and you wish to construct a 95% confidence interval about the result. Mathematically, the number of people we would need to poll is

$$n = \frac{z_{\alpha/2}^2 P_b(1 - P_b)}{E^2}$$

where P_b is the proportion of the population that prefers candidate A and E is the margin of error. We replaced this formula with

$$n = \frac{z_{\alpha/2}^2 \cdot 0.25}{E^2}.$$

a. Why do we use this new formula instead of the first formula and why are we justified in using 0.25 in the numerator?

   Solution: We use this formula because P_b is unknown ($\hat{P}_b$ is unknown too before the poll is conducted) so we made $P_b = 0.5$ to maximize the value of n to be conservative.

b. Find the value of n assuming the margin of error is $\pm 5\%$. You can use the fact that $z_{\alpha/2} = 1.96$ for $\alpha = 0.05$.

   Solution: We get $n = 385$. (The formula yields 384.16 but we round this up since we cannot have a fraction of a person and we thus use the next largest integer).

Problem 2. (5 points.) Suppose you compute the mean of a random sample of size $n = 100$ and obtain $\bar{X} = 25$. The standard deviation is not known but is estimated to be 1.5. Construct a 95% confidence interval for the true mean (μ) of the data. You may leave your answer in terms of $t_{\alpha/2}$ or $z_{\alpha/2}$ (as appropriate) but you should specify the value of α.

Solution: We find the confidence interval is

$$\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 25 \pm t_{\alpha/2} \cdot 0.15$$

where $s = 1.5$ is the estimated standard deviation and $\alpha = 0.05$.

Problem 3. (5 points.) Suppose you obtain samples from a normal (Gaussian) distribution with unknown mean and unknown variance. Your samples values are 2.1, 0.9, 1.8, 2.5 and 2.3. Construct a 95% confidence interval for the mean. You may use either of the facts (as appropriate) that for $\alpha = 0.05$, $z_{\alpha/2} = 1.96$ and $t_{\alpha/2} = 2.776$ for $n = 5$.

Solution: We find the sample mean is $\bar{X} = 1.92$ and the sample variance is $s = 0.6261$. Thus, the 95% confidence interval for the mean is

$$\bar{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 1.92 \pm 2.776 \cdot 0.28 = 1.92 \pm 0.777.$$