EE 503

Quiz 10 Solution

Fall 2019, 15 Minutes, 15 Points

Problem 1. (10 points.) Let X be binomially distributed with $n = 49$ and $p = \frac{1}{2}$. Using the CLT find

a. $P(X = 25)$.

Solution: X has mean $np = 24.5$ and variance $npq = 12.25$ so let Y be normally distributed with mean $\mu = 24.5$ and variance $\sigma^2 = 12.25$. Then with Z denoting standard normal we find

$$\begin{aligned} P(X=25) &= P(24.5 \le Y \le 25.5) = P\left(\frac{24.5-\mu}{\sigma} \le Z \le \frac{25.5-\mu}{\sigma}\right) \\ &= P(0 \le Z \le 0.2857) = \Phi(0.2857) - \Phi(0) \\ &= \Phi(0.2857) - 0.5. \end{aligned}$$

b. $P(X \le 30)$.

Solution: We find

$$\begin{aligned} P(X \le 30) &= P(Y \le 30.5) = P\left(Z \le \frac{30.5-\mu}{\sigma}\right) \\ &= P(Z \le 1.7143) = \Phi(1.7143). \end{aligned}$$

Note: You must write your answers in terms of the standard normal (Gaussian) cdf defined as

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2} dz.$$

Problem 2. (5 points.) Suppose the random variables $X_i,\ i = 1, 2, ..., n$ are uncorrelated and have the same mean μ and variance σ^2. Define the sample mean $\bar{X}$ and the sample variance $\bar{V}$ as

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad \bar{V} = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2.$$

The sample variance is written as a function of n deviates, where $(X_i - \bar{X})$ is a deviate. Show that $\bar{V}$ can be written as a function of $(n-1)$ deviates.

Solution: We find

$$\bar{V} = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 = \frac{1}{n-1}\left[(X_i - \bar{X})^2 + \sum_{i=2}^{n}\left(X_i - \bar{X}\right)^2\right]$$

Now, $\sum_{i=1}^{n}(X_i - \bar{X}) = 0$ so $(X_i - \bar{X}) = -\sum_{i=2}^{n}(X_i - \bar{X})$. Thus,

$$\bar{V} = \frac{1}{n-1}\left[\left(\sum_{i=2}^{n}(X_i - \bar{X})\right)^2 + \sum_{i=2}^{n}\left(X_i - \bar{X}\right)^2\right]$$

which is a function of $(n-1)$ deviates.