

# EE 503

## Quiz 4 Solution

Fall 2019, 15 Minutes, 15 Points

**Problem 1.** (8 points.) A random variable  $X$  has pdf

$$f(x) = \begin{cases} \frac{1}{\beta}e^{-(x-c)/\beta}, & x \geq c, \\ 0, & \text{elsewhere,} \end{cases}$$

where  $\beta$  and  $c$  are positive real numbers.

a. Find the cumulative distribution function (cdf) of  $X$ .

**Solution:**

$$\begin{aligned} F(x) &= \int_{-\infty}^x f(t)dt \\ &= \int_c^x \frac{1}{\beta}e^{-(t-c)/\beta}dt \end{aligned}$$

which becomes

$$F(x) = \begin{cases} 1 - e^{-(x-c)/\beta}, & x \geq c, \\ 0, & \text{elsewhere.} \end{cases}$$

b. Find  $P(X > 2c)$ .

**Solution:**

$$P(X > 2c) = 1 - F(2c) = e^{-c/\beta}.$$

**Problem 2.** (7 points.) Suppose the random variable  $X$  has a cdf

$$F(x) = \begin{cases} 1 - e^{-x}, & x \geq 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that  $X^2 - 4X + 3$  is negative.

**Solution:** We observe

$$X^2 - 4X + 3 < 0 \Leftrightarrow (X - 1)(X - 3) < 0 \Leftrightarrow 1 < X < 3$$

so

$$P(X^2 - 4X + 3 < 0) = P(1 < X < 3) = F(3) - F(1) = e^{-1} - e^{-3} = 0.3181.$$