

EE 503

Quiz 3 Solution

Fall 2019, 15 Minutes, 15 Points

You must show the appropriate mathematical expressions to receive credit.

Problem 1. (7 points.) Suppose there are events A and B such that

$$P(B) = 0.2, P(B|A) = 0.3, P(A \cup B) = 0.4.$$

Find $P(A|B)$.

Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{P(A \cap \bar{B}) + P(A \cap B)}.$$

$$P(A \cap \bar{B}) = P(A \cup B) - P(B) = 0.4 - 0.2 = 0.2.$$

Thus,

$$0.3 = P(B|A) = \frac{P(A \cap B)}{0.2 + P(A \cap B)} \Rightarrow P(A \cap B) = \frac{0.06}{0.7}.$$

Hence,

$$P(A|B) = \frac{0.06/0.7}{0.2} = 0.4286.$$

Problem 2. (8 points.) Suppose you have 10 red balls and 5 blue balls in a container. You pick a ball at random and then replace that ball and 2 additional balls of the same color picked back into the container. Now, pick a ball a second time.

- a. Find the probability that the second choice is red.

Solution:

$$\begin{aligned} P(R_2) &= P(R_2|R_1)P(R_1) + P(R_2|B_1)P(B_1) \\ &= \frac{12}{17} \cdot \frac{10}{15} + \frac{10}{17} \cdot \frac{5}{15} = 0.6667. \end{aligned}$$

- b. Find the probability that the second choice is the same color as the first choice you picked before replacing it.

Solution:

$$\begin{aligned}P(R_2 \cap R_1) &= P(R_2|R_1)P(R_1) = \frac{12}{17} \cdot \frac{10}{15} = \frac{120}{255} \\P(B_2 \cap B_1) &= P(B_2|B_1)P(B_1) = \frac{7}{17} \cdot \frac{5}{15} = \frac{35}{255}.\end{aligned}$$

Thus,

$$P(\text{same color}) = P(R_2 \cap R_1) + P(B_2 \cap B_1) = 0.6078.$$