EE 503

Quiz 1 Solution

Fall 2019, 15 Minutes, 15 Points

Problem 1. (7 points.) Let F be a $\sigma-$field of subsets of Ω. Suppose that $B \in F$. Is

$$G = \{ \ A \cap B : A \in F\}$$

a $\sigma-$field of subsets of B? Justify your answer mathematically. Note that this problem is asking if G is a $\sigma-$field of subsets of B (not Ω).

Solution: We must show that G satisfies the definition of a $\sigma-$field.

i. $\emptyset \in F$ therefore $\emptyset = \emptyset \cap B \in G$.

ii. If $A_1, A_2, \ldots \in F$ then $\bigcup_i (A_i \cap B) = (\bigcup_i A_i) \cap B \in G$.

iii. If $A \in F$ then $\bar{A} \in F$ so $\overline{A \cap B}$ (where this last complement is taken inside B) is $B \setminus (A \cap B) = \bar{A} \cap B \in G$.

Thus, G is a $\sigma-$field.

Problem 2. (8 points.) Suppose we define $d : \mathbf{X} \times \mathbf{X} \to \mathbf{R}$ by

$$d(x, y) = |1/x - 1/y|$$

where $\mathbf{X}$ is the set of positive real numbers and $x,\ y$ are points in $\mathbf{X}$. Is d a metric on $\mathbf{X}$? Justify your answer mathematically.

Solution: We must show that d satisfies the definition of a metric.

i. $d(x, y) = |1/x - 1/y| \geq 0 \ \forall x, y \in \mathbf{X}$ and $d(x, y) = 0 \Leftrightarrow x = y$.

ii. $d(x, y) = d(y, x) \ \forall x, y \in \mathbf{X}$.

iii.

$$\begin{aligned} d(x,z) &= |1/x - 1/z| = |1/x - 1/y + 1/y - 1/z| \\ &\leq |1/x - 1/y| + |1/y - 1/z| = d(x, y) + d(y, z) \ \forall x, y, z \in \mathbf{X}. \end{aligned}$$

Thus, d is a metric on $\mathbf{X}$.