

EE 503

Homework 14

(Not to be handed in for grading)

Work all 3 problems.

Problem 1. Let $X = (X_1, X_2, \dots, X_n)$ be i.i.d. where each $X_i \sim N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. Find the MLE for μ and σ^2 .

Problem 2. Text 11.10 (modified). An information source generates i.i.d. bits X_n for which $P(X_n = 0) = P(X_n = 1) = 1/2$.

- Suppose X_n is transmitted over a binary symmetric channel (BSC) with probability of error $= p$ (a BSC just means it is just as likely a 0 is received as a 1 as it is a 1 is received as a 0). Find the probabilities of the possible outputs of the channel.
- Suppose X_n is transmitted over K consecutive identical and independent BSCs. Does the sequence of channel outputs form a Markov chain?
- Find the K -step transition probabilities that relate the input bits from the source to the output bits of the K th channel.
- What are the probabilities of the possible outputs of the K th channel as $K \rightarrow \infty$?

Problem 3. Here we consider a Heuristic estimator. Suppose you have available to you outputs of an algorithm that produces a random variable that has a noncentral chi-square distribution with two degrees of freedom with mean and variance

$$\begin{aligned} E[X] &= 2\sigma^2 + s^2 \\ \text{Var}[X] &= 4\sigma^4 + 4\sigma^2 s^2. \end{aligned}$$

You have available to you n samples of this distribution. You wish to estimate s^2 . Can you think of a way to do this from observing the mean and variance expressions?