EE 503

Homework 12

Due Wednesday November 20, 2019 at 6 p.m.

Work all 2 problems.

Problem 1. Let S be a set and n a natural number. Recall

$$\limsup_{n\to\infty} A_n = \cap_{n=1}^{\infty} \cup_{k=n}^{\infty} A_k.$$

Show

$$\limsup_{n\to\infty} A_n = \{x \in S : x \in A_n \text{ for infinitely many } n\}.$$

Problem 2. Let $X_k,\ k = 1, 2, \ldots$ be a sequence of independent normal (Gaussian) random variables with mean 0 and variance σ^2. Define

$$Y_n = \frac{1}{N} \sum_{k=n-N+1}^{n} X_k, \quad n = 0, 1 \ldots$$

where we take $Y_n = 0$ for $n < 0$.

This kind of operation of averaging is often used in signal processing. It is a type of low pass filter (LPF) since if the X_k had sinusoidal components they would average out (that is, be filtered out). We will refer to this type of filter as an I&D (integrate and dump) filter.

Now for implementation purposes we can also construct a LPF to approximate the above averaging filter using an IIR (infinite impulse response) filter as

$$\tilde{Y}_n = (1-\alpha)X_n + \alpha \tilde{Y}_{n-1}, \quad n = 0, 1 \ldots$$

where we take $\tilde{Y}_{-1} = 0$ and $\alpha \in (0, 1)$.

a. Show that $E[\tilde{Y}_n] = E[Y_n] = 0$ for any $\alpha \in (0, 1)$.

b. Determine the value of $\alpha = \alpha(N)$ so that the variance of the IIR filter output ($\tilde{Y}_n$) matches the variance of the I&D filter output (Y_n) as $n \to \infty$.