EE 503

Homework 11

Due Wednesday November 13, 2019 at 6 p.m.

Work all 5 problems.

Problem 1. Suppose ω is selected at random in the interval [0, 1]. For each of the following state whether the sequence of random variables converges surely, almost surely or not at all. If the sequence does converge indicate the random variable or constant to which the sequence converges.

a. $X_n(\omega) = \dfrac{\omega}{n}$.

b. $Y_n(\omega) = \omega\left(1 - \dfrac{1}{n}\right)$.

c. $Z_n(\omega) = \omega e^n$.

d. $V_n(\omega) = \omega^n$.

e. $W_n(\omega) = \cos^n 2\pi\omega$.

Problem 2. Let X_n be a sequence of i.i.d. equiprobable Bernoulli random variables and let

$$Y_n = 2^n X_1 X_2 \ldots X_n.$$

a. Show this sequence converges almost surely and indicate the limit.

b. Determine whether or not Y_n converges in the mean square sense.

Problem 3. A random variable X is said to be Laplacian with parameter $\alpha > 0$ if it has density

$$f_X(x) = \frac{\alpha}{2}e^{-\alpha|x|}, \quad -\infty < x < \infty.$$

Let X_n be a sequence of Laplacian random variables with parameter $\alpha = n$. Show this sequence converges in probability (and hence in distribution).

Problem 4. We know that convergence in probability always implies convergence in distribution but the converse is not, in general, true. However, suppose the random sequence Z_n converges to Z in distribution where Z is some constant z_0. In this case show that $Z_n \to Z$ in distribution implies $Z_n \to Z$ in probability.

Problem 5. A particular Cauchy random variable X has density

$$f(x) = \frac{1}{\pi} \frac{1}{1 + x^2}, \quad -\infty < x < \infty.$$

X does not have a mgf since its moments do not exist. Show, in fact, that $E[X]$ does not exist by trying to analytically compute the mean of X.