EE 503
Homework 10
Due Wednesday November 6, 2019 at 6 p.m.

Work all 4 problems.

Problem 1. Suppose the random variables X_i, $i = 1, 2, ..., n$ are uncorrelated and have the same mean μ and variance σ^2. Define the sample mean $\bar{X}$ as

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

and the sample variance $\bar{V}$ as

$$\bar{V} = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \bar{X} \right)^2 .$$

Show

    a. $E\left(\bar{X}\right) = \mu$.

    b. $Var\left(\bar{X}\right) = \sigma^2/n$.

    c. $E\left(\bar{V}\right) = \sigma^2$.

Problem 2. Suppose the random variable X is normally distributed with mean 2 and variance 9. Find

    a. $P(X < 5)$.

    b. $P(X > -1)$.

    c. $P(-1 < X < 5)$.

    d. $P(X < 10)$.

Problem 3. Let X be binomially distributed with $n = 50$ and $p = \frac{1}{3}$. Find

    a. $P(X = 17)$.

    b. $P(X \leq 20)$.

Problem 4. We can think of X in Problem 3 as the sum of 50 Bernoulli random variables with success probability $p = \frac{1}{3}$. So let us use the CLT to approximate our answers in Problem 3. Using the CLT find

a. $P(X = 17)$.

b. $P(X \leq 20)$.

Remember to use continuity correction as we discussed in our class example.