EE 503

Homework 9

Due Wednesday October 30, 2019 at 6 p.m.

Work all 5 problems.

Problem 1. Let $X \sim N(\mu_1, \sigma_1^2)$. Find a, b such that if $Y = aX + b$ then $Y \sim N(\mu_2, \sigma_2^2)$.

Problem 2. Let X and Y have joint *pdf*

$$f_{XY}(x,y) = \begin{cases} 2e^{-(x+y)}, & 0 \le y \le x < \infty \\ 0, & \text{elsewhere.} \end{cases}$$

Let $Z = X + Y$. Show Z has *pdf*

$$f_Z(z) = ze^{-z}u(z).$$

Note: X and Y are not independent.

Problem 3. Consider the random variable X with the Pareto density

$$f(x) = \begin{cases} \lambda x^{-\lambda-1}, & x > 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Let $Y = \ln(X)$ (the natural log). Find the density function for Y.

Problem 4. Let Z_1 and Z_2 be independent standard normal random variables. Let $Y_1 = Z_1 + Z_2$ and let $Y_2 = Z_2 - Z_1$.

a. Show that Y_1 and Y_2 are independent. *Hint*: You can compute the joint characteristic function (or the joint moment generating function) for Y_1 and Y_2 and show that it factors (no integration is necessary).

b. Find the joint density function for Y_1 and Y_2. *Hint*: Make use of independence.

Problem 5. Let Z_1 and Z_2 be independent random variables each having an exponential density of the form $f_Z(z) = \lambda e^{-\lambda z}U(z)$. Define $X = Z_2$, $Y = Z_2(1 + Z_1)$. Find

a. Find $E(Y|X = x)$.

b. Find $E(E(Y|X))$.

c. Find $Var(E(Y|X))$.

d. Find $Var(Y|X = x)$.

e. Find $E(Var(Y|X))$.

f. Find the best MSE predictor of Y given $X = x$.

g. Find the best linear MSE predictor of Y based on X.