EE 503

Homework 8

Due Wednesday October 23, 2019 at 6 p.m.

Work all 7 problems.

Problem 1. Suppose X and Y are independent random variables, each normally distributed with mean 0 and variance σ^2.

Compute $E[|X - Y|]$.

Problem 2. The two-dimensional continuous random variable (X, Y) has joint *pdf*

$$f_{XY}(x,y) = \begin{cases} x^2 + \dfrac{xy}{3}, & 0 \le x \le 1,\ 0 \le y \le 2, \\ 0, & \text{elsewhere.} \end{cases}$$

a. Compute $f_{X|Y}(x|y)$.

b. Find $E[X|Y = y]$.

Problem 3. Suppose the two-dimensional continuous random variable (X, Y) has joint *pdf*

$$f_{XY}(x,y) = \begin{cases} x + y, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

a. Find $f_{X|Y}(x|y)$.

b. Let $B = \{X + Y \ge 1/4\}$. Find $P(B)$.

Problem 4. Suppose we have two independent random variables X and Y with respective densities

$$f_X(x) = \begin{cases} e^{-x}, & x > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

and

$$f_Y(y) = \begin{cases} y/2, & 0 < y < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Let $W = X/Y$. Find the density function for W.

Problem 5. Let X be a normally distributed random variable with mean 1 and variance 1. Suppose $Y = f(X)$, i.e., Y is a function of X. It is known that $E(Y) = 5$ and $Var(Y) = 29$. Furthermore, $r_{XY} = 1$, i.e., the correlation coefficient between X and Y is 1. Find the function f.

Problem 6. Suppose the two-dimensional ranndom variable (X, Y) has desnity

$$f(x,y) = \begin{cases} x^3 + \dfrac{xy^2}{\alpha}, & 0 < x < 1,\ 0 < y < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

a. Show that the value of α that makes this a valid density function is $\alpha = 8/3$.

b. Compute $P(X + Y \geq 2)$.

Problem 7. Let Ω be an uncountable set and let

$$F = \left\{ E \subseteq \Omega : E \text{ or } \overline{E} \text{ is at most countable} \right\}.$$

Show F is a $\sigma-$algebra (or $\sigma-$field).