EE 503

Homework 6

Due Wednesday October 9, 2019 at 6 p.m.

Work all 6 problems.

Problem 1. Let X be a geometric random variable, i.e.,

$$P(X = k) = (1-p)^{k-1}p, \ k = 1, 2, \dots$$

Find the conditional distribution function $F_X(x|A)$ where A is the event

a. $A = \{X > s\}$.

b. $A = \{X < s\}$.

c. $A = \{X \text{ is even}\}$.

Problem 2. Suppose $P(X = k) = \beta(1-\beta)^{k-1}, \ 0 < \beta < 1, \ k = 1, \ 2, \ 3, \dots$. Find Var(X).

Problem 3. The continuous random variable X has pdf

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Find Var(X).

Problem 4. The continuous random variable X has pdf

$$f(x) = \begin{cases} 3x^2, & 0 \le x \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Let $Y = X^4$. Find Var(Y).

Remark: You do not need to find the pdf of Y to work this problem.

Problem 5. A man is saving up to buy a new car at a cost of N units of money. He starts with k units of money $(0 < k < N)$ and tries to win the remainder with the following gamble with his bank manager. He tosses a fair coin; if it turns up heads the bank manager pays him one unit of money, but if it comes up tails then he pays the manager one unit of money. He keeps tossing the coin and playing this game until either he has won enough units of money to buy the car or he loses his k units of money (goes bankrupt).

Let A_k denote the event that he is eventually bankrupt after his initial capital was k units. Let $p_k = P(A_k)$.

a. Show $p_k = \frac{1}{2}(p_{k+1} + p_{k-1})$ if $0 < k < N$.

b. The result in part (a) is a linear difference equation subject to the boundary conditions $p_0 = 1,\ p_N = 0$. Solve this difference equation for p_k. Hint: If you have not solved this type of equation analytically before then you can instead proceed directly as follows: First let $b_k = p_k - p_{k-1}$. Show $b_k = b_{k-1}$ and thus $b_k = b_1$ for all k. Continue from here.

Problem 6. Suppose the discrete random variable X has probability mass function

$$P(X = x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!}, & x = 0, 1, 2, \ldots,\ \lambda \geq 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the moment generating function of X and use it to compute $E[X]$ and $Var(X)$.