EE 503

Homework 4

Due Wednesday September 25, 2019 at 6 p.m.

Work all 6 problems.

Problem 1. Suppose that f and g are probability density functions (pdf's) defined on the same interval $[a,\ b]$.

a. Show that $f + g$ is not a valid pdf on the same interval.

b. For any number $\beta,\ 0 < \beta < 1,$ show that $\beta f(x) + (1-\beta)g(x)$ is a valid pdf on the interval $[a,\ b]$.

Problem 2. Let X be the life length of an electronic device (measured in hours). Suppose that X is a continuous random variable with *pdf*

$$f(x) = k/x^n,\ 2,000 \leq x \leq 10,000.$$

a. For $n = 2$, determine k.

b. For $n = 3$, determine k.

c. For general n, determine k.

d. For general n, find the probability that the device will reach the end of its life before 5000 hours have elapsed.

Problem 3. If the random variable K is uniformly distributed over (0, 5), what is the probability that the roots of the equation $4x^2 + 4Kx + K + 2 = 0$ are real.

Problem 4. Suppose that the random variable X has possible values 1, 2, 3,... and that

$$P(X = r) = k(1-\beta)^{r-1},\ 0 < \beta < 1.$$

a. Determine the constant k.

b. Find the *mode* of this distribution, i.e., find that value of r which makes $P(X = r)$ maximum.

Problem 5. Say someone has two coins and tells you that one of the coins has 2 heads and the other coin is fair (you cannot see the coins).

a. Suppose the person flips the coins one after another. Let $X = 1$ if the first toss is heads and $X = 0$ if the first toss is tails. Let $Y = 1$ if the second toss is heads and let $Y = 0$ if the second toss is tails. Are X and Y independent random variables? Explain your answer.

b. Suppose the person flips just one of the coins and it turns up heads. Based only on this information can you say which coin is most likely the fair coin? If so, justify your answer. If not, explain why.

Problem 6. Text 4.17 (modified). A random variable X has pdf

$$f(x) = \begin{cases} c\,(1 - x^2)\,, & -1 \le x \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

a. Find c.

b. Find the cumulative distribution function (cdf) of X.

c. Find $P(0.25 < X < 0.5)$.