EE 503

Homework 3

Due Wednesday September 18, 2019 at 6 p.m.

Work all 9 problems.

Problem 1. Suppose $A \subset B$ with $P(A) = 1/4$ and $P(B) = 1/3$.

a. Find $P(A|B)$.

b. Find $P(B|A)$.

Problem 2. Show that $P(AB|C) = P(A|BC)P(B|C)$.

Problem 3. Show that $P(ABC) = P(A|BC)P(B|C)P(C)$.

Problem 4. Box 1 contains 1 White and 999 Red balls and Box 2 contains 1 Red and 999 White balls. Suppose someone picks a box at random and then selects a ball from that box and the ball picked is Red. Find the probability the Red ball came from Box 1.

Problem 5. Suppose there are events A and B such that

$$P(B) = 0.2,\ P\left(B|A\right) = 0.4,\ P\left(A \cup B\right) = 0.5.$$

Find $P\left(A|B\right)$.

Problem 6. Suppose A and B are events such that $A \cap B = \phi$. Can A and B be independent? If not, explain why not. If so, explain how.

Problem 7. Suppose you have R red balls and B blue balls in a container. You pick a ball at random and then replace that ball and C balls of the same color back into the container. Now, pick a ball a second time. Find the probability that the second choice is red.

Problem 8. Suppose I have two coins. One is fair so $P(H) = P(T) = 1/2$. The other coin has $P(H) = 1/3$ and $P(T) = 2/3$. Suppose I pick one of the coins at random and give it to you. You flip the coin twice and it comes up heads both times. Based on this information find the probability that I gave you the fair coin.

Problem 9. Let X be the set of positive integers. Let S consist of all the subsets E of X such that either E or $\overline{E}$ is finite. Show that S is an algebra but not a σ-algebra.