EE 503

Homework 2 Solution

Instructor: Christopher Wayne Walker, TA: James Huang

Problem 1. Ordering a "deluxe" pizza from a certain restaurant means you have 4 choices from 15 available toppings. How many combinations are possible if toppings can be repeated? If they cannot be repeated? Note the order in which the 4 toppings are selected does not matter so in both cases we have sampling without order.

Solution:
If the toppings cannot be repeated, then there are $\binom{15}{4} = 1365$ cases. If the toppings can be repeated, it means that $x_1 + x_2 + \cdots + x_{15} = 4$ where x_i is a non-negative integer denoting the count of topping i that is put on the pizza. Thus there are $\binom{18}{4} = 3060$ cases.

Problem 2. A certain deck of cards contains 10 red cards numbered 1 to 10 and 10 black cards numbered 1 to 10. How many ways are there of arranging the 20 cards in a row? Suppose we draw the cards at random and lay them in a row. What is the probability that red and black cards alternate?

Solution:
There are 20! ways of arranging the cards in a row. The probability that the row of cards has alternative red and black cards is

$$\frac{2 \cdot 10! \cdot 10!}{20!} = 1.0825 \times 10^{-5}.$$

Problem 3. An urn contains 3 red balls, 5 white balls and 4 black balls. Three balls are chosen at random. What is the probability of choosing 2 red balls if

a. the sampling is done without replacement?

b. the sampling is done with replacement?

Solution:

a. $\frac{\binom{3}{2}\binom{9}{1}}{\binom{12}{3}} = 0.1227.$

b. $\frac{\binom{3}{2} \cdot 3 \cdot 3 \cdot 9}{12^3} = 0.1406.$

Problem 4. Suppose you are dealt a hand in poker as discussed in class. Let A be the event getting 5 cards of the same suit (this is called a flush). Let B be the event getting a pair (exactly 2 of the same face value). Let C be the event getting a full house (exactly 3 of one face value and exactly 2 of another face value).
For event A you should exclude the probability of a royal flush (this is 5 cards of the same suit with face values Ace, King, Queen, Jack, 10) and also exclude the probability of a straight flush (this is 5 cards of the same suit with consecutive face values but excludes the royal flush). An example of a straight flush is Jack,10,9,8,7 all of the same suit and another example is 5,4,3,2,Ace all of the same suit where in this latter case the Ace has a face value of 1.
Find

a. $P(A)$.

b. $P(B)$.

c. $P(C)$.

In each case show the expression you used to get your answer.

Solution:

a. $P(A) = \frac{\binom{4}{1} \cdot \binom{13}{5} - 40}{\binom{52}{5}} = 0.0020.$

b. $P(B) = \frac{\binom{13}{1} \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4^3}{\binom{52}{5}} = 0.4226.$

c. $P(C) = \frac{\binom{13}{1} \cdot \binom{4}{3} \cdot \binom{12}{1} \cdot \binom{4}{2}}{\binom{52}{5}} = 0.0014.$

Alternatively, this kind of problem can also be solved with order. For example

$$P(C) = \frac{\binom{5}{3} \cdot \binom{2}{2} \cdot 52 \cdot 3 \cdot 2 \cdot 48 \cdot 3}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48},$$

where the denominator is simply ordering 5 cards out of 52 cards, and $\binom{5}{3} \cdot \binom{2}{2}$ means that you choose 3 seats for one face value and 2 seats for another.