EE 503

Homework 1

Due Wednesday September 4, 2019 at 6 p.m.

Work all 9 problems.

Problem 1. Two components C_1 and C_2 of a system are tested and declared to be in one of three possible states: F, functioning; R, not functioning but repairable; and W, worthless.

a. Find the sample space for this experiment.

b. What is the set corresponding to the event "none of the components are worthless"?

Problem 2. Three balls numbered 1 to 3 in an urn are drawn at random one at a time until the urn is empty. The sequence of the ball numbers is noted.

a. Find the sample space for this experiment.

b. Find the sets A_k corresponding to the events "ball number k is selected in the kth draw," for $k = 1,\ 2,\ 3$.

c. Find the set $A_1 \cap A_2 \cap A_3$ and describe the event in words.

d. Find the set $A_1 \cup A_2 \cup A_3$ and describe the event in words.

d. Find the set $(A_1 \cup A_2 \cup A_3)^c$ and describe the event in words.

Problem 3. Let the events A and B have $P(A) = x$, and $P(B) = y$, and $P(A \cap B) = z$. Use Venn diagrams to find the following:

a. $P(A^c \cup B^c)$.

b. $P(A \cap B^c)$.

c. $P(A^c \cup B)$.

d. $P(A^c \cap B^c)$.

Problem 4. Prove the following identities:

a. $\binom{n}{k} = \binom{n}{n-k}$

b. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Problem 5. If $\mathbf{X} = \{1, 2, 3, 4, 5\}$ find the smallest algebra (or field) that contains the sets $\{1\}$ and $\{2,\ 3,\ 5\}$.

Problem 6. Let $x,\ y$ be points in $\mathbf{R}$. Show that

$$d(x, y) = |x - y|$$

is a metric on $\mathbf{R}$.

Problem 7. Let $a = (x_1,\ y_1),\ b = (x_2,\ y_2)$ be points in $\mathbf{R}^2$. Show that

$$d(a, b) = \max\{|x_1 - x_2|,\ |y_1 - y_2|\}$$

is a metric on $\mathbf{R}^2$. This $d(a, b)$ is often denoted $d_\infty(a, b)$.

Problem 8. Let $d(x, y)$ be a metric on $\mathbf{X}$. Show that

$$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$$

is also a metric on $\mathbf{X}$.

Problem 9. We know that if we have n students and n chairs we can seat the students in $n!$ ways (just the number of ways of ordering n items). Explain what is wrong with the following argument that derives a different answer.

We first select a student which can be done in n ways. We then select a chair for this student which can also be done in n ways. Thus the first student is selected and seated in n^2 ways. Next, we select the second student in $(n-1)$ ways and her chair in $(n-1)$ ways so the selection and seating of the second student can be done in $(n-1)^2$ ways. Continuing on in this way, we select and seat the n students in $n^2(n-1)^2(n-2)^2 \cdots 3^2 2^2 1^2 = (n!)^2$ ways.