EE 464

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Lecture Notes Part 6

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6.0 Independence of Events

Definition: Events $A_1, \dots,\ A_n$ are *independent* if for any k-tuple $(1 \le k \le n)$ we have

$$P(A_{i_1} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \cdots P(A_{i_k})$$

where

$$i_j \in \{1, 2 \dots, n\} \quad \text{and} \quad i_m \neq i_s,\ m \neq s.$$

Special Case: A_1, A_2 are independent if $P(A_1 \cap A_2) = P(A_1)P(A_2)$. Recall,

$$P(A|B) = \begin{cases} \dfrac{P(A \cap B)}{P(B)}, & P(B) > 0 \\ 0, & P(B) = 0. \end{cases}$$

Assuming $P(B) > 0$ then if A, B are independent it follows that

$$P(A|B) = \frac{P(A)P(B)}{P(B)} = P(A).$$

Similarly, for A, B independent

$$P(B|A) = P(B).$$

Claim: If A, B are independent then so are

i. A and $\overline{B}$.

ii. $\overline{A}$ and B.

iii. $\overline{A}$ and $\overline{B}$

A proof of (i) will be given in class.

Remarks: If A, B, C are independent then

i. $P(A|B) = P(A), \quad P(A|(B \cap C)) = P(A), \quad P(A|(B \cup C)) = P(A).$

ii. Ω is independent of every event, $\emptyset$ is independent of every event.

iii. If event A is contained in event B $(A \subset B)$ and $0 < P(A) < 1$, $0 < P(B) < 1$, then A and B are not independent.

A proof of (iii) will be given in class.

Repeated Independent Trials

Suppose event A from an experiment has $P(A) = p$. Suppose we repeat the experiment n times in such a way that the result of the experiment each time is independent of the other times. Then,

i. $P(A \text{ occurs every time}) = p^n$.

ii. $P(A \text{ never occurs}) = (1-p)^n$.

iii. $P(A \text{ occurs at least once}) = 1 - (1-p)^n$.

Now let X be the number of trials until A occurs for the first time. Then X is random.

i. $P(X = 1) = p$.

ii. $P(X \geq k) = (1-p)^{k-1}$.

iii. $P(X = k) = P(X \geq k) - P(X \geq k+1)$

$$= (1-p)^{k-1} - (1-p)^k = (1-p)^{k-1}p.$$

A communications example will be given in class.