EE 464

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Lecture Notes Part 2

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2.0 Set Operations and Notation

Definition: A *set* is a collection of elements.

Examples:

i. $\mathbf{Z} = \{\text{all integers}\}$.

ii. $\mathbf{R} = \{\text{all real numbers}\}$.

iii. Let $\mathbf{A} = \{-2, 3, 7\}$. Then $\mathbf{A}$ is a *subset* of $\mathbf{Z}$. We write $\mathbf{A} \subset \mathbf{Z}$ or $\mathbf{A} \subseteq \mathbf{Z}$. The number 3 is an *element* of $\mathbf{A}$. We write $3 \in \mathbf{A}$.

Definition: The *empty set* is the set that contains no elements. We write $\{\emptyset\}$ or just $\emptyset$ for the empty set.

Suppose we have a set $\mathbf{S}$ with three elements: $\mathbf{S} = \{a, b, c\}$. What are the possible subsets of $\mathbf{S}$. They are $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, c\}$, $\emptyset$.

Note that the set itself and the empty set are always subsets of the given set.

Remark: Given a set $\mathbf{S}$ with n elements, there are 2^n subsets of $\mathbf{S}$.

Definition: The family of all subsets of a set $\mathbf{X}$ is denoted $P(\mathbf{X})$ and is called the *power set* of $\mathbf{X}$.

Let $\mathbf{S}$ be any set and let $\mathbf{A}, \mathbf{B}$ be two subsets of $\mathbf{S}$. Then $\mathbf{A} \cup \mathbf{B}$ denotes the union of $\mathbf{A}$ and $\mathbf{B}$ and $\mathbf{A} \cap \mathbf{B}$ denotes the intersection of $\mathbf{A}$ and $\mathbf{B}$.

i. If $\mathbf{D} = \mathbf{A} \cup \mathbf{B}$, then $\mathbf{D}$ is also a subset of $\mathbf{S}$, i.e., $\mathbf{D} \subset \mathbf{S}$ and $d \in \mathbf{D}$ *iff* (if and only if) $d \in \mathbf{A}$ <u>or</u> $d \in \mathbf{B}$. Some authors write $\mathbf{A} + \mathbf{B}$ for $\mathbf{A} \cup \mathbf{B}$.

ii. If $\mathbf{E} = \mathbf{A} \cap \mathbf{B}$, then $\mathbf{E} \subset \mathbf{S}$ and $e \in \mathbf{E}$ *iff* $e \in \mathbf{A}$ <u>and</u> $e \in \mathbf{B}$. Some authors write $\mathbf{AB}$ for $\mathbf{A} \cap \mathbf{B}$.

iii. **Definition:** $\mathbf{A}^C$ or $\overline{\mathbf{A}}$ is called the *complement* of $\mathbf{A}$ and consists of all those elements of $\mathbf{S}$ that are not in $\mathbf{A}$. More specifically, we say $\overline{\mathbf{A}}$ is the *complement* of $\mathbf{A}$ *in* $\mathbf{S}$ but this is shortened when there is no confusion as to what the encompassing set is.

iv. **General Notes:**

$$\mathbf{A} \cup \overline{\mathbf{A}} = \mathbf{S}, \quad \mathbf{A} \cap \overline{\mathbf{A}} = \emptyset, \quad \overline{\overline{\mathbf{A}}} = \mathbf{A}, \quad \mathbf{A} \cap \emptyset = \emptyset, \quad \mathbf{A} \cup \emptyset = \mathbf{A},$$

$$\overline{\mathbf{S}} = \emptyset, \quad \overline{\emptyset} = \mathbf{S}, \quad \mathbf{A} \cap \mathbf{S} = \mathbf{A}, \quad \mathbf{A} \cup \mathbf{S} = \mathbf{S}.$$

Venn diagrams are often useful for ascertaining truths and properties of sets and their operations. These will be demonstrated in class.

Demorgan's Rules:

$$\overline{\mathbf{A} \cup \mathbf{B}} = \overline{\mathbf{A}} \cap \overline{\mathbf{B}}, \quad \overline{\mathbf{A} \cap \mathbf{B}} = \overline{\mathbf{A}} \cup \overline{\mathbf{B}}.$$