EE 464

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Lecture Notes Part 13

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13.0 Introduction to Random Processes

Here we will introduce some convergence concepts.

Consider the infinite sequence of random variables

$$X_1, X_2, \ldots.$$

Recall X_i is really $X_i(\omega)$, a function of ω.

i. $X_n \to X$ everywhere if $X_n \to X$ as $n \to \infty$.

ii. $X_n \to X$ almost everywhere if $\{\omega \in \Omega : X_n(\omega) \to X(\omega)\}$ exists and has probability 1.

iii. $X_n \to X$ in probability if for any $\epsilon > 0$ $P(|X_n - X| > \epsilon) \to 0$ as $n \to \infty$.

iv. $X_n \to X$ in mean-square sense if $E\left(|X_n - X|^2\right) \to 0$ as $n \to \infty$.

v. $X_n \to X$ in distribution (or Law) if $\lim_{n\to\infty} P\left(X_n \le x\right) = P\left(X \le x\right).$

Convergence in mean-square implies convergence in probability. To see this note by Cauchy-Schwarz

$$P(|X_n - X| > \epsilon) \le \frac{E\left(|X_n - X|^2\right)}{\epsilon^2} \to 0 \text{ as } n \to \infty.$$