

# EE 464

## Homework 7

Do not hand it.

**Work all 9 problems.**

**Problem 1.** Leon-Garcia Ch.4 problem 34.

**Problem 2.** Leon-Garcia Ch.4 problem 59.

**Problem 3.** Leon-Garcia Ch.4 problem 60.

**Problem 4.** Leon-Garcia Ch.5 problem 22.

**Problem 5.** Suppose the random variables  $X_i$ ,  $i = 1, 2, \dots, n$  are uncorrelated and have the same mean  $\mu$  and variance  $\sigma^2$ . Define the sample mean  $\bar{X}$  as

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

and the sample variance  $\bar{V}$  as

$$\bar{V} = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2.$$

Show

- a.  $E(\bar{X}) = \mu$ .
- b.  $Var(\bar{X}) = \sigma^2/n$ .
- c.  $E(\bar{V}) = \sigma^2$ .

For the following problem note that if

$$X \sim N(\mu, \sigma^2)$$

then

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1).$$

**Problem 6.** Suppose the random variable  $X$  is normally distributed with mean 2 and variance 9. Find

- a.  $P(X < 5)$
- b.  $P(X > -1)$
- c.  $P(-1 < X < 5)$
- d.  $P(X < 10)$ .

**Problem 7.** Suppose  $W$  and  $Z$  are two independent random variables each uniformly distributed over the interval  $(1, 2)$ . Let  $X = W$  and  $Y = WZ$ .

- a. Find the best mean square error (MSE) predictor of  $Y$  given  $X = x$ .
- b. Find the best linear MSE predictor of  $Y$  based on  $X$ .

**Problem 8.** The two-dimensional continuous random variable  $(X, Y)$  has joint *pdf*

$$f_{XY}(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \leq x \leq 1, 0 \leq y \leq 2, \\ 0, & \text{elsewhere.} \end{cases}$$

- a. Compute  $f_{X|Y}(x|y)$ .
- b. Find  $E[X|Y = y]$ .

**Problem 9.** Let  $Z_1$  and  $Z_2$  be independent random variables each having an exponential density of the form  $f_Z(z) = \lambda e^{-\lambda z} U(z)$ . Define  $X = Z_2$ ,  $Y = Z_2(1 + Z_1)$ . Find

- a. Find  $E(Y|X = x)$ .
- b. Find  $E(E(Y|X))$ .
- c. Find  $Var(E(Y|X))$ .
- d. Find  $Var(Y|X = x)$ .
- e. Find  $E(Var(Y|X))$ .
- f. Find the best MSE predictor of  $Y$  given  $X = x$ .
- g. Find the best linear MSE predictor of  $Y$  based on  $X$ .