

# EE 464

## Homework 5

Due Monday April 7, 2003

**Work all 15 problems.**

**Problem 1.** Leon-Garcia Ch.3 problem 53.

**Problem 2.** Leon-Garcia Ch.3 problem 57.

**Problem 3.** Leon-Garcia Ch.3 problem 59.

**Problem 4.** Leon-Garcia Ch.3 problem 67.

**Problem 5.** Leon-Garcia Ch.3 problem 72.

**Problem 6.** Suppose  $X$  is uniform in the interval  $(0, 1)$ . Find the density function for  $Y = -\ln X$ .

**Problem 7.** The continuous random variable  $X$  has *pdf*

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Let  $Y = e^{-X}$ . Find the *pdf* of  $Y$ .

**Problem 8.** Suppose  $X \sim N(0, 1)$ . Let  $Y = X^2$ . Find the *pdf* of  $Y$ .

**Problem 9.** Suppose  $X$  is uniform over the interval  $(-1, 1)$ . Let  $W = |X|$ . Find the *pdf* of  $W$ .

**Problem 10.** Suppose  $P(X = k) = \beta(1 - \beta)^{k-1}$ ,  $0 < \beta < 1$ ,  $k = 1, 2, 3, \dots$

a. Find  $E(X)$ .

b. Find  $\text{Var}(X)$ .

**Problem 11.** The continuous random variable  $X$  has *pdf*

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

- a. Find  $E(X)$ .
- b. Find  $\text{Var}(X)$ .

**Problem 12.** The continuous random variable  $X$  has *pdf*

$$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Let  $Y = X^4$ .

- a. Find  $E(Y)$ .
- b. Find  $\text{Var}(Y)$ .

*Remark:* You do not need to find the *pdf* of  $Y$  to work this problem.

**Problem 13.** Tchebycheff's inequality.

- a. Use Tchebycheff's inequality to bound  $P(|X - \mu| \geq \frac{3}{2}\sigma)$ .
- b. Now assume that  $X \sim U(1 - 1/\sqrt{3}, 1 + 1/\sqrt{3})$ . Compute analytically  $P(|X - \mu| \geq \frac{3}{2}\sigma)$ .

*Remark:* This problem shows that if you know the density of a random variable you can get a better estimate on probability bounds than that provided by Tchebycheff's inequality.

**Problem 14.** Suppose  $X \sim U(a, b)$ . Find the moment generating function  $M_X(s)$ . Specify any restrictions on  $s$  in order that  $M_X(s)$  exists.

**Problem 15.** Say  $P(X = k) = (1-p)^{k-1}p$ ,  $k = 1, 2, 3, \dots$ ,  $0 < p < 1$ . Find the moment generating function  $M_X(s)$  and use it to calculate the moments  $E(X)$  and  $E(X^2)$  and then find the variance of  $X$  using these two moments. Specify any restrictions on  $s$  in order that  $M_X(s)$  exists.