EE 464

Homework 2

Due Monday Feburary 10, 2003

Work all 16 problems.

Problem 1. Leon-Garcia Ch.2 problem 18.

Problem 2. Leon-Garcia Ch.2 problem 31.

Problem 3. Leon-Garcia Ch.2 problem 36.

Problem 4. Leon-Garcia Ch.2 problem 46.

Problem 5. Leon-Garcia Ch.2 problem 60.

Problem 6. Let the events A and B have $P(A) = x$, and $P(B) = y$, and $P(A \cap B) = z$. Use Venn diagrams to find the following:

   a. $P(A^c \cup B^c)$.

   b. $P(A \cap B^c)$.

   c. $P(A^c \cup B)$.

   d. $P(A^c \cap B^c)$.

Problem 7. An urn contains 3 red balls, 5 white balls and 4 black balls. Three balls are chosen at random. What is the probability of choosing 2 red balls if

   a. the sampling is done without replacement?

   b. the sampling is done with replacement?

Repeat parts (a) and (b) for the probability of choosing 2 red balls and 1 white ball.

Problem 8. A fair coin is tossed until the first head comes up.

    a. Find the sample space, Ω, for this experiment.

    b. Find the probability that the first head appears on the kth toss.

    c. Verify that $P(\Omega) = 1$.

       Let A be the event "$k > 4$." Let B be the event "$k > 10$."

    d. Find $P(A)$.

    e. Find $P(B)$.

    f. Find $P(B^c)$.

    g. Find $P(A \cap B)$.

    h. Find $P(A \cup B)$.

Problem 9. Suppose that from N objects we choose k at random with replacement. What is the probability that no object is chosen more than once? (You may assume that $k < N$.)

Problem 10. There are n persons in a room.

    a. What is the probability that at least two persons have the same birthday? (You may ignore leap years and assume the year has 365 days.)

    b. Calculate this probability for $n = 50$.

    c. How large need n be for this probability to be greater than 0.5?

Problem 11. Show that

$$P(AB|C) = P(A|BC)P(B|C).$$

Problem 12. Suppose there are events A and B such that

$$P(B) = 0.2, \ P(B|A) = 0.4, \ P(A \cup B) = 0.5.$$

Find $P(A|B)$.

Problem 13. Suppose A and B are events such that $A \cap B = \phi$ (they are mutually exclusive). Can A and B be independent? If not, explain why not. If so, explain how.

Problem 14. Suppose you have R red balls and B blue balls in a container. You pick a ball at random and then replace that ball and C balls of the same color back into the container. Now pick a ball a second time. Find the probability that the second choice is red.

Problem 15. Suppose I have two coins. One is fair so $P(H) = P(T) = 1/2$. The other coin has $P(H) = 1/3$ and $P(T) = 2/3$. Suppose I pick one of the coins at random and give it to you. You flip the coin twice and it comes up heads both times. Based on this information find the probability that I gave you the fair coin.

Problem 16. Consider the following information concerning the diagnostic testing for some disease. There is a

- probability of 0.98 of the test being positive when the subject has the disease.

- probability of 0.95 of the test being negative when the subject does not have the disease

- probability of a person having the disease is $1/1000 = 0.001$.

a. Find the probability that a randomly chosen person tests positive for the disease.

b. Find the probability that a person has the disease given that the test shows positive for the disease.