EE464: HOMEWORK 1 SOLUTIONS

Problem 1. Solution

a) $S = \{1, 2, 3, 4, 5, 6\}$
b) $A = \{2, 4, 6\}$
c) $A^c = \{1, 3, 5\}$ "odd number of dots"

Problem 2. Solution

The outcome of this experiment consists of a pair of numbers (x,y) where x = number of dots in first toss and so on and y =number of dots in second toss. Therefore, S = set of ordered pairs (x, y) where $x, y \in \{1, 2, 3, 4, 5, 6\}$which are listed in the table below.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Checkmarks indicate elements of events

b)

√		√		√	
	√		√		√
√		√		√	
	√		√		√
√		√		√	
	√		√		√

c)

	√		√		√
	√		√		√
	√		√		√

d) B is a subset of A so when B occurs then A also occurs. thus B implies A.

e) $A \bigcap B^C$ = "sum is even and both tosses show odd number".

√		√		√	
√		√		√	
√		√		√	

f) C = "number of dots differ by 1"

	√				
√		√			
	√		√		
		√		√	
			√		√
				√	

Comparing the tables for A and C we see that C="number of dots differ by 1"

$$A \cap C = \phi$$

Problem 3. Solution

a) S = set of ordered pairs (x_1, x_2) where x_1 specifies state of C_1

	x_2=F	x_2=R	x_2=K
x_1=F	(F,F)	(F,R)	(F,K)
x_1 =R	(R,F)	(R,R)	(R,K)
x_1=K	(K,F)	(K,R)	(K,K)

b) $\{(F,F),(F,R),(R,F),(R,R)\}$

Problem 4. Solution

a) $S = \{(1,2,3),(2,1,3),(3,1,2),(1,3,2),(2,3,1),(3,2,1)\}$

b)
$A_1 = \{(1,2,3),(1,3,2)\}$
$A_2 = \{(1,2,3),(3,2,1)\}$
$A_3 = \{(1,2,3),(2,1,3)\}$

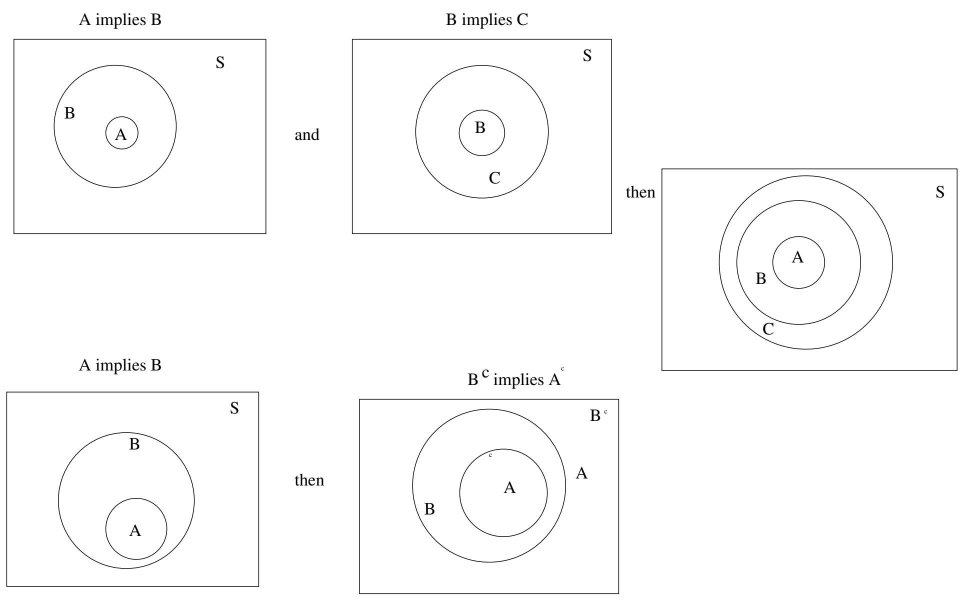
c) $A_1 \cap A_2 \cap A_3 = \{(1,2,3)\}$
"the number of every ball corresponds to the number of the draw"

d) $A_1 \cup A_2 \cup A_3 = \{(1,2,3),(1,3,2),(3,2,1),(2,1,3)\}$
"the number of at least one ball corresponds to the number of the draw"

e) $(A_1 \cup A_2 \cup A_3)^c = \{(3,1,2),(2,3,1)\}$
"none of the balls has a number that corresponds to the number of the draw"

Problem 5. Solution

Venn diagrams are shown below



Problem 6. Solution

a) the difference between the two outcomes can be 0,1,2,3,4,5. So the sample space $S = \{0,1,2,3,4,5\}$.

b) $A = \{0,2,4,6\}$

Problem 7. Solution

The event "A or B" is $A \cup B$.

the event “not both A and B“ is $(A \cap B)^c$

Therefore the event “A or B but not both” is $(A \cup B) \cap (A \cap B)^c = (A \cap B^c) \cup (A^c \cap B) = A \Delta B$

Problem 8. Solution

$X = \{1, 2, 3, 4\}$

Lets list down the sets that *have to be* in the sigma field F which contains $\{1\}, \{2, 3\}$.

$\{1, 2, 3, 4\}, \phi$.... since the whole space and its complement have to be in the sigma field.

$\{1\}, \{2, 3\}, \{2, 3, 4\}, \{1, 4\}$... since complements have to be in the sigma field.

$\{1, 4\} \cap \{2, 3, 4\} = \{4\}$

$\{X, \phi, \{1\}, \{4\}, \{2, 3\}, \{1, 2, 3\}, \{1, 4\}, \{2, 3, 4\}\}$.... one can verify that this family is an algebra. Since all its members have to be in the sigma algebra that contains $\{1\}, \{2, 3\}$, we cann't have a smaller algebra. Therefore it is the smallest algebra.

Problem 9. Solution

One can verify that the first three axioms of metric are satisfied.

To prove triangle inequality, let's consider third point $c = (x_3, y_3)$.

First consider the inequality

for any $P_1, P_2, Q_1, Q_2 \in \mathbb{R}$

$(P_1 + Q_1)^2 + (P_2 + Q_2)^2 = |P_1^2 + 2P_1P_2 + P_2^2| + |Q_1^2 + 2Q_1Q_2 + Q_2^2|$

$= P_1^2 + 2|P_1P_2| + P_2^2 + Q_1^2 + 2|Q_1Q_2| + Q_2^2 \leq (\sqrt{P_1^2 + P_2^2} + \sqrt{Q_1^2 + Q_2^2})^2$

This can be proved using Schwart's inequality.

Now substitute $P_1 = (x_1 - x_2)$, $P_2 = (y_1 - y_2)$ and $Q_1 = (x_2 - x_3)$, $Q_2 = (y_2 - y_3)$ gives,

$(x_1 - x_3)^2 + (y_1 - y_3)^2 \leq \left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} + \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}\right)^2$

taking square root on both sides,

$\sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} \leq \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} + \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$

Therefore, the triangle inequality is also satisfied.

Hence $d(.)$is a metric.

Problem 10. First three conditions for metric can be easily verified.

Also the triangle inequality is satisfied. Since for a third point $c(x_3, y_3)$,

$d(a, c) = |x_1 - x_3| + |y_1 - y_3| = |x_1 - x_2 + x_2 - x_3| + |y_1 - y_2 + y_2 - y_3|$

$\leq |x_1 - x_2| + |x_2 - x_3| + |y_1 - y_2| + |y_2 - y_3|$

Therefore $d(.)$ is a metric.