EE 464 Exam 1 Solutions

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Inst: C.W. Walker

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	16	
Total	100	

Instructions and Information:

1) Print your name and location at the top of the page.

2) Make sure your exam has 8 problems and 14 numbered pages. Pages 9-11 are for extra workspace (reference them in the given problem if you use them – do not tear these out). Do not write any work to be graded on the back of the pages.

3) This is a closed book exam. A formula sheet is provided at the end of the exam. You may use a calculator. **You have 75 minutes to take this exam**.

4) Partial credit will be given but you must **show your work to receive any credit**.

5) **Circle or box your final answers**.

Problem 1. Let $\Omega = \{a,\ b,\ c\}$.

a. Find the subsets of Ω that one would need to add (if any) to the following list of subsets in order that the list constitutes a sigma field.

$$\phi,\ \{a,\ b,\ c\},\ \{a,\ b\},\ \{b,\ c\}.$$

Solution: $\{a\},\ \{b\},\ \{c\},\ \{a,\ c\}$.

b. Consider the following sigma field:

$$F = \phi,\ \{a,\ b,\ c\},\ \{a\},\ \{b,\ c\}.$$

Suppose someone tells you the probability assignments for these events are

$$P(\{\phi\}) = 0,\ P(\{a,\ b,\ c\}) = 1,\ P(\{a\}) = .3,\ P(\{b,\ c\}) = .6$$

Explain why this is not a valid probability measure.

Solution: $P(\{a\}) + P(\{b,\ c\}) = .3 + .6 = .9 \neq 1 = P(\{a,\ b,\ c\})$.

Problem 2. There are 6 white balls and 3 black balls in an urn.

a. Three of the balls are chosen at random. Find the probability that exactly 2 black balls are chosen if the balls are chosen without replacement.
Solution:

$$\frac{\binom{3}{2}\binom{6}{1}}{\binom{9}{3}} = \frac{3}{14} = 0.214.$$

b. Determine the number of distinguishable ways these 9 balls could be placed in a row.

Solution: $\dfrac{9!}{6!\ 3!} = 84$.

Problem 3. Box 1 contains 1 white and 99 red balls. Box 2 contains 1 red and 99 white balls. A ball is picked from a randomly selected box. If the ball picked is red what is the probability that it came from box 1?

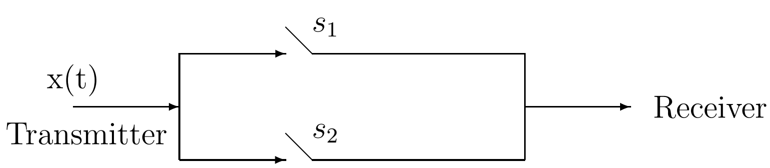
Solution:

$$P(\text{Box1}|\text{Red}) = \frac{P(\text{Red}|\text{Box1})P(\text{Box1})}{P(\text{Red})}.$$

$$\begin{aligned}P(\text{Red}) &= P(\text{Red}|\text{Box1})P(\text{Box1}) + P(\text{Red}|\text{Box2})P(\text{Box2})\\ &= \frac{99}{100}\cdot\frac{1}{2} + \frac{1}{100}\cdot\frac{1}{2} = \frac{1}{2}.\end{aligned}$$

$$P(\text{Box1}|\text{Red}) = \frac{\frac{99}{100}\cdot\frac{1}{2}}{\frac{1}{2}} = \frac{99}{100}.$$

Problem 4. Consider the transmission of a signal as shown in the following diagram.



A signal is transmitted along two paths as shown. In each path there is a switch to pass through. The probability that switch 1 is closed and allows the signal to pass through is $p_1 = 0.5$. The probability that switch 2 is closed and allows the signal to pass through is $p_2 = 0.7$. The probability that both switches are closed at the same time is 0.35. The signal transmission is successful if the signal $x(t)$ sent at the transmitter reaches the receiver along either or both paths.

Find the probability that the transmission is successful.

Solution: $P(\text{success}) = 0.5 + 0.7 - 0.35 = 0.85.$

Problem 5. Recall that in a game of poker there are 52 cards. The face value of a card can be either 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A and the suit of a card can be Hearts (red), Spades (black), Diamonds (red) or Clubs (black). Suppose you are dealt 5 cards.

Find the probability that you get a triple, i.e., exactly 3 cards with the same face value. You do not have to numerically simplify your answer but do explain why you use each factor that you use in your probability expression.

Solution:

$$P(\text{triple}) = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}4^2}{\binom{52}{5}} = 0.021$$

where

$\binom{13}{1}$ is the number of ways to choose the face value for the triple

$\binom{4}{3}$ is the number of ways to choose the suits for the 3 cards

$\binom{12}{2}$ is the number of ways to choose the face value for the other 2 cards

4^2 is the number of ways to choose the suits for the other 2 cards

Problem 6. Suppose there are events A and B such that

$$P(B) = 0.3,\ P(B|A) = 0.2,\ P(A \cup B) = 0.6.$$

Find $P(A|B)$.

Solution:

$$P(A \cap B) = P(B|A)P(A) = 0.2P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow 0.6 = P(A) + 0.3 - 0.2P(A)$$

so

$$0.3 = 0.8P(A) \Rightarrow P(A) = 3/8 = 0.375.$$

Thus

$$P(A|B) = \frac{0.2P(A)}{P(B)} = \frac{0.2(0.375)}{0.3} = 0.25.$$

Problem 7.

a. Find $P(A|B)$ if $A \cap B = \emptyset$.

   Solution: If $P(B) \neq 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\emptyset)}{P(B)} = 0.$$

   If $P(B) = 0$ then we still have $P(A|B) = 0$ by definition.

b. Suppose now that $P(A|B) > P(A)$. Show that $P(B|A) > P(B)$.

   Solution:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} > P(A) \Rightarrow \frac{P(B|A)}{P(B)} > 1 \Rightarrow P(B|A) > P(B).$$

Problem 8. Alice and Bob agree to take turns rolling a die to see who gets to go to Disneyland. Since Alice is youngest she gets to roll first. If she gets a 6 she wins and the contest is over. Otherwise, Bob gets to go next and if he gets a 6 he wins and the contest is over. Otherwise, the process starts all over again with Alice rolling first and then Bob unless Alice wins. This process continues until someone wins.

Find the probability that Alice gets to go to Disneyland. Note: A die has six sides so the probability of rolling a 6 is 1/6 on each roll.

Solution:

$$\begin{aligned} P(A \text{ wins}) &= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \cdots \\ &= \frac{1}{6}\left[a^0 + a^1 + a^2 + a^3 + \cdots\right] \end{aligned}$$

where $a = \left(\frac{5}{6}\right)^2$. Thus,

$$P(A \text{ wins}) = \frac{1}{6} \cdot \frac{1}{1-a} = \frac{1}{6} \cdot \frac{1}{1 - 25/36} = \frac{6}{11} = 0.545.$$