Name: ________________________ **Viewing Location:** ____________________

EE 464 Exam 1

Feb. 19, 2003

Inst: C.W. Walker

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	16	
Total	100	

Instructions and Information:

1) Print your name and location at the top of the page.

2) Make sure your exam has 8 problems and 14 numbered pages. Pages 9-11 are for extra workspace (reference them in the given problem if you use them – do not tear these out). Do not write any work to be graded on the back of the pages.

3) This is a closed book exam. A formula sheet is provided at the end of the exam. You may use a calculator. **You have 75 minutes to take this exam**.

4) Partial credit will be given but you must **show your work to receive any credit**.

5) **Circle or box your final answers**.

Problem 1. Let $\Omega = \{a,\ b,\ c\}$.

a. Find the subsets of Ω that one would need to add (if any) to the following list of subsets in order that the list constitutes a sigma field.

$$\phi,\ \{a,\ b,\ c\},\ \{a,\ b\},\ \{b,\ c\}.$$

b. Consider the following sigma field:

$$F = \phi,\ \{a,\ b,\ c\},\ \{a\},\ \{b,\ c\}.$$

Suppose someone tells you the probability assignments for these events are

$$P(\{\phi\}) = 0,\ P(\{a,\ b,\ c\}) = 1,\ P(\{a\}) = .3,\ P(\{b,\ c\}) = .6$$

Explain why this is not a valid probability measure.

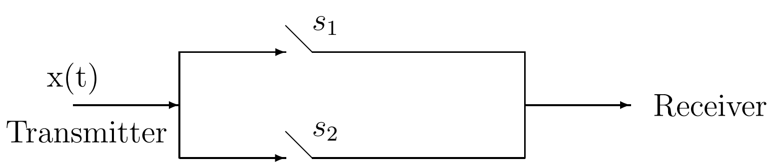
Problem 2. There are 6 white balls and 3 black balls in an urn.

a. Three of the balls are chosen at random. Find the probability that exactly 2 black balls are chosen if the balls are chosen without replacement.

b. Determine the number of distinguishable ways these 9 balls could be placed in a row.

Problem 3. Box 1 contains 1 white and 99 red balls. Box 2 contains 1 red and 99 white balls. A ball is picked from a randomly selected box. If the ball picked is red what is the probability that it came from box 1?

Problem 4. Consider the transmission of a signal as shown in the following diagram.



A signal is transmitted along two paths as shown. In each path there is a switch to pass through. The probability that switch 1 is closed and allows the signal to pass through is $p_1 = 0.5$. The probability that switch 2 is closed and allows the signal to pass through is $p_2 = 0.7$. The probability that both switches are closed at the same time is 0.35. The signal transmission is successful if the signal $x(t)$ sent at the transmitter reaches the receiver along either or both paths.

Find the probability that the transmission is successful.

Problem 5. Recall that in a game of poker there are 52 cards. The face value of a card can be either 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A and the suit of a card can be Hearts (red), Spades (black), Diamonds (red) or Clubs (black). Suppose you are dealt 5 cards.

Find the probability that you get a triple, i.e., exactly 3 cards with the same face value. You do not have to numerically simplify your answer but do explain why you use each factor that you use in your probability expression.

Problem 6. Suppose there are events A and B such that

$$P(B) = 0.3,\ P(B|A) = 0.2,\ P(A \cup B) = 0.6.$$

Find $P(A|B)$.

Problem 7.

a. Find $P(A|B)$ if $A \cap B = \emptyset$.

b. Suppose now that $P(A|B) > P(A)$. Show that $P(B|A) > P(B)$.

Problem 8. Alice and Bob agree to take turns rolling a die to see who gets to go to Disneyland. Since Alice is youngest she gets to roll first. If she gets a 6 she wins and the contest is over. Otherwise, Bob gets to go next and if he gets a 6 he wins and the contest is over. Otherwise, the process starts all over again with Alice rolling first and then Bob unless Alice wins. This process continues until someone wins.

Find the probability that Alice gets to go to Disneyland. Note: A die has six sides so the probability of rolling a 6 is 1/6 on each roll.

Extra workspace. If you use this space for work to be graded reference it from the given problem.

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Exam 1 Notes

Demorgan's Rules: $\overline{\mathbf{A} \cup \mathbf{B}} = \overline{\mathbf{A}} \cap \overline{\mathbf{B}}, \quad \overline{\mathbf{A} \cap \mathbf{B}} = \overline{\mathbf{A}} \cup \overline{\mathbf{B}}.$

Definition: The *sample space* Ω is the set of all possible outcomes of a random experiment.

Definition: An *event* is a (particular kind of) subset of the sample space.

Rules for Events

i. $\emptyset$ and Ω are events.

ii. If $\mathbf{A}$ and $\mathbf{B}$ are events then so are $\mathbf{A} \cap \mathbf{B}$, $\mathbf{A} \cup \mathbf{B}$, $\mathbf{B} \backslash \mathbf{A}$ and $\overline{\mathbf{A}} = \Omega \backslash \mathbf{A}$.

iii. If $A_1,\ A_2, \ldots,$ are events then so are

$$A_1 \cup A_2 \cup \cdots = \bigcup_{n=1}^{\infty} \mathbf{A}_n \quad \text{and} \quad A_1 \cap A_2 \cap \cdots = \bigcap_{n=1}^{\infty} \mathbf{A}_n.$$

Remark $\mathbf{A} \triangle \mathbf{B} := (\mathbf{A} \backslash \mathbf{B}) \cup (\mathbf{B} \backslash \mathbf{A})$ (symmetric difference).

Definition: Let $\mathbf{X}$ be a nonempty set. An *algebra* of sets on $\mathbf{X}$ is a nonempty collection $A \in P(\mathbf{X})$ which is closed under finite unions and complements, i.e.,

i. if $E_1,\ E_2, \ldots,\ E_n \in A$ then $\bigcup_{k=1}^{n} E_k \in A$.

ii. if $E \in A$ then $\overline{E} \in A$.

Definition: A *σ-algebra* (or *σ-field*) is an algebra which is closed under countable unions. So, a collection F of subsets of $\mathbf{X}$ is called a σ-field if

i. $\emptyset \in F$.

ii. $A_1,\ A_2, \ldots \in F \Rightarrow \bigcup_{k=1}^{\infty} A_k \in F$.

iii. $A \in F \Rightarrow \overline{A} \in F$.

Definition: If $(\mathbf{X}, d)$ is any metric space, the σ-field generated by the family of open sets in $\mathbf{X}$ is called the *Borel* σ-field, denoted by $B_{\mathbf{X}}$. Its members are called Borel sets.

Special case: When $\mathbf{X} = \mathbf{R}$ (the real line), we have $B_{\mathbf{R}}$.

Definition: A *probability measure* P on (Ω, F) is a function

$$P : F \to [0,\ 1]$$

that maps

$$A \mapsto P(A)$$

that satisfies the following axioms of probability:

i. $P(\emptyset) = 0, \quad P(\Omega) = 1, \quad 0 \leq P(A) \leq 1$ (redundant)

ii. If $A_1,\ A_2, \ldots$ is a sequence of pairwise disjoint events then

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots.$$

The triple $(\Omega,\ F,\ P)$ is called a *probability space*.

Definition: ${}_nP_n$ is the total number of ways of arranging or permuting n different objects. Note:

$${}_nP_n = n!.$$

Definition: ${}_nP_r$ is the total number of ways of permuting r of n objects. Note:

$${}_nP_r = \frac{n!}{(n-r)!}.$$

Definition: C is the number of ways of choosing r of n objects disregarding order.

$$C = \binom{n}{r} = \frac{n!}{(n-r)!r!}.$$

The *binomial theorem*:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

Permutations when not all objects are different: Recall we can arrange n different objects in $n!$ ways. Say, we have n_1 of one kind, n_2 of another,$\ldots$, n_k of the kth kind such that $n_1 + n_2 + \cdots + n_k = n$. As will be explained in class, the number of permutations of these n objects is

$$\frac{n!}{n_1! n_2! \cdots n_k!}.$$

Definition: The *conditional probability* of event B occurring given that event A occurs is defined to be

$$P(B|A) = \begin{cases} \dfrac{P(B \cap A)}{P(A)}, & P(A) > 0 \\ 0, & P(A) = 0. \end{cases}$$

Another way of writing this is

$$P(B|A) = \begin{cases} \dfrac{P(A|B)P(B)}{P(A)}, & P(A) > 0 \\ 0, & P(A) = 0. \end{cases}$$

Let Ω be a sample space and assume $P(A) > 0$. Given a partition $B_1,\ B_2, \ldots, B_k$ of the sample space Ω, i.e., events $B_1,\ B_2, \ldots, B_k$ such that $P(B_i) > 0$, $i = 1, 2, \ldots, k$, $B_i \cap B_j = \emptyset$, $i \neq j$ and $\bigcup_{i=1}^{k} B_i = \Omega$ then (law of total probability)

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \cdots + P(A|B_k)P(B_k)$$

and (Bayes' Rule)

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \cdots + P(A|B_k)P(B_k)}.$$

A Sum Formula:

$$\sum_{k=M}^{\infty} a^k = \frac{a^M}{1-a}, \quad 0 < a < 1.$$