EE 564

Homework 8

Due Monday April 14, 2014

Work all 3 problems.

Problem 1. In class we had a model of a PLL and said the closed-loop transfer function was

$$H(s) = \frac{KG(s)/s}{1 + KG(s)/s}$$

where G(s) was the transfer function of the loop filter inside the PLL. Derive the above expression for $H(s)$ given our model.

Problem 2. In class we indicated that $H(s)$ for the PLL mentioned in Problem 1 could be written as

$$H(s) = \frac{(2\zeta\omega_n - \omega_n^2/K)s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

The (one-sided) noise-equivalent bandwidth of the loop is defined as

$$B_{eq} = \frac{1}{G}\int_0^\infty |H(f)|^2 df$$

where $G = \max |H(f)|^2$. Show that

$$B_{eq} = \frac{1 + (\tau_2\omega_n)^2}{8\zeta/\omega_n}$$

where we have utilized

$$G(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s}$$

and

$$\omega_n = \sqrt{K/\tau_1}, \quad \zeta = \omega_n(\tau_2 + 1/K)/2.$$

Problem 3. Suppose the input to the PLL is a signal of the form

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

and it is corrupted by additive narrowband noise

$$n(t) = x(t) \cos 2\pi f_c t - y(t) \sin 2\pi f_c t$$

where $x(t)$ and $y(t)$ are independent stationary Gaussian noise processes with (two-sided) power spectral density $N_0/2$ W/Hz. This may be written as

$$n(t) = n_i(t) \cos[2\pi f_c t + \phi(t)] - n_q(t) \sin[2\pi f_c t + \phi(t)]$$

where

$$\begin{aligned} n_i(t) &= x(t) \cos \phi(t) + y(t) \sin \phi(t) \\ n_q(t) &= -x(t) \sin \phi(t) + y(t) \cos \phi(t). \end{aligned}$$

After $s(t) + n(t)$ is multiplied by the output of the VCO and the double frequency terms are neglected (since the loop filter filters these out) show that we get the error signal

$$e(t) = A_c \sin \Delta\phi + n_i(t) \sin \Delta\phi - n_q(t) \cos \Delta\phi$$

where $\Delta\phi = \hat{\phi} - \phi$.