EE 564

Homework 6 Solutions

Problem 1. In class we looked at CPFSK signaling and we showed the time-varying phase of the carrier is

$$\phi(t;I) = 4\pi T f_d \int_{-\infty}^{t} \left[\sum_n I_n g(\tau - nT) \right] d\tau$$

where I_n denotes the sequence of amplitudes obtained by mapping $k-$bit blocks of binary digits from the information sequence a_n into amplitude levels $\pm 1, \pm 3, \ldots, \pm(M-1)$, and $g(t)$ is a rectangular pulse of amplitude $1/2T$ and duration T seconds. By performing the above integration show that

$$\begin{aligned} \phi(t;I) &= 2\pi f_d T \sum_{k=-\infty}^{n-1} I_k + 2\pi f_d q(t-nT) I_n \\ &= \theta_n + 2\pi h I_n q(t-nT) \end{aligned}$$

where

$$\begin{aligned} h &= 2 f_d T \\ \theta_n &= \pi h \sum_{k=-\infty}^{n-1} I_k \\ q(t) &= \begin{cases} 0, & t < 0 \\ t/2T, & 0 \le t \le T \\ 1/2, & t > T. \end{cases} \end{aligned}$$

Solution: The phase of the carrier in the interval $nT \le t \le (n+1)T$ is found by integrating $\phi(t;I)$. Note that

$$q(t) = \int_0^t g(\tau) d\tau$$

so

$$q(t-nT) = \int_0^{t-nT} g(\tau) d\tau.$$

Since we are computing the phase for $nT \leq t \leq (n+1)T$ then $t - nT \geq 0$ and $t - (n-1)T > 0$ so we can write

$$\begin{aligned}
\phi(t; I) &= 4\pi T f_d \int_{-\infty}^{t} \left[\sum_n I_n g(\tau - nT) \right] d\tau \\
&= 4\pi T f_d \int_{-\infty}^{t} \left[\sum_{k=-\infty}^{n-1} I_k g(\tau - kT) + I_n g(\tau - nT) \right] d\tau \\
&= 2\pi T f_d \sum_{k=-\infty}^{n-1} I_k + 4\pi T f_d I_n q(\tau - nT)
\end{aligned}$$

which is the desired result.

Problem 2. Explain clearly, using text and/or mathematics, how OQPSK (SQPSK) avoids 180 degree phase discontinuities.

Solution: In OQPSK the in-phase and quadrature pulse streams are staggered and thus do not change at the same time. Since only one component can make a transition at one time the phase changes are limited to $0°$ and $\pm 90°$ every T seconds whereas with QPSK the phase can change by $0°$, $\pm 90°$ and $180°$ every $2T$ seconds.

Problem 3. Suppose that one of two equally likely messages is to be transmitted over an AWGN channel. The signal vectors are

$$s_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad s_1 = \begin{pmatrix} -2 \\ -2 \end{pmatrix}.$$

corresponding to messages m_0 and m_1, respectively. Draw the decision boundaries and give the decision logic for the optimum receiver, that is, give the optimum rule for deciding m_0 or m_1 was sent.

Solution: Since the channel is AWGN and the messages are equally likely the optimum receiver uses the minimum distance metric (see figure below). Let $r^T = (r_1, r_2)$. Then the decision rule becomes

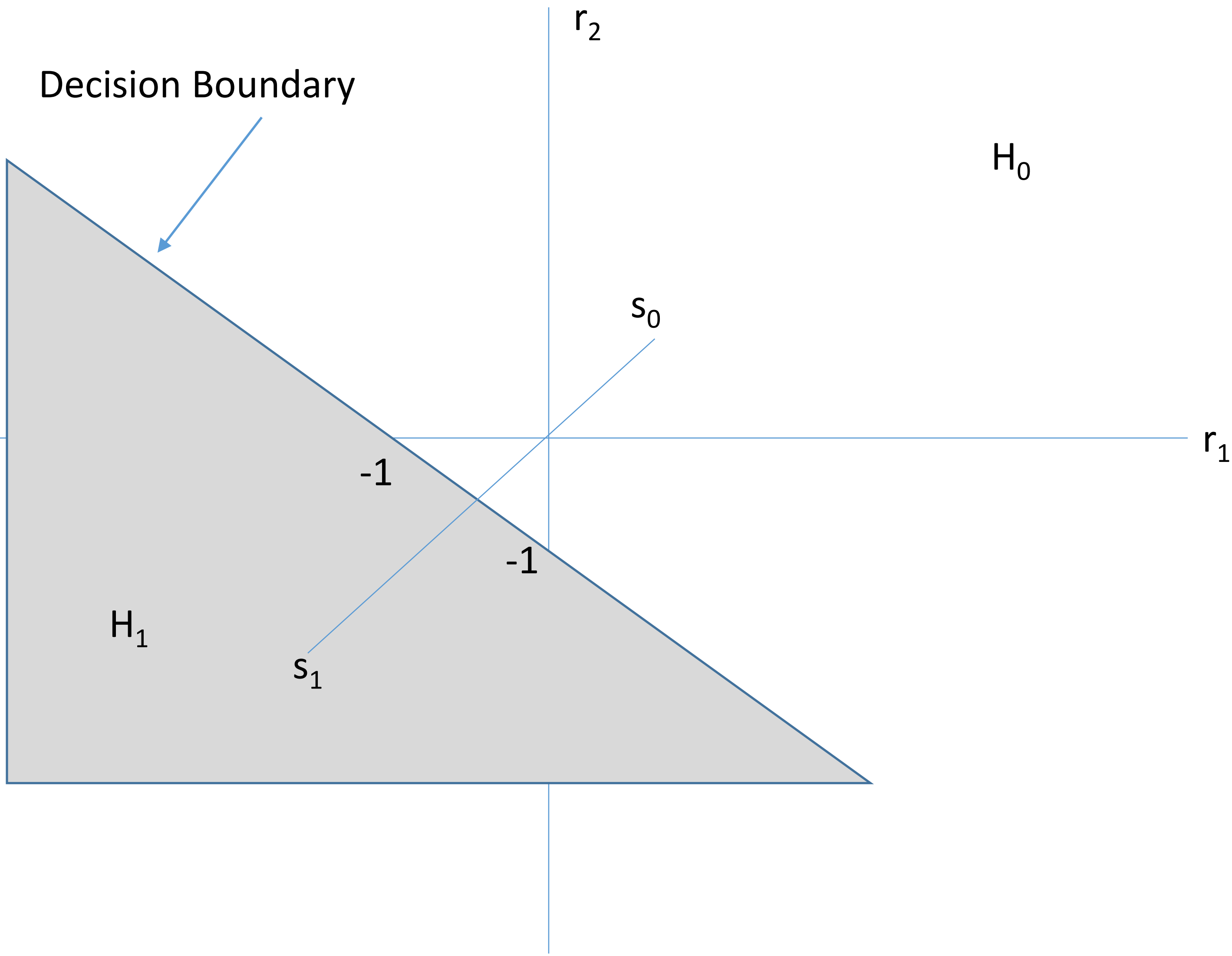
$$2r^T s_0 - |s_0|^2 \underset{m_1}{\overset{m_0}{\gtrless}} 2r^T s_1 - |s_1|^2$$

or

$$2(r_1 - r_2) - 2 \underset{m_1}{\overset{m_0}{\gtrless}} 2(-2r_1 - 2r_2) - 8$$

which becomes

$$r_1 + r_2 \underset{m_1}{\overset{m_0}{\gtrless}} -1.$$



HW6 Problem 3