# EE 564

## Homework 4

Due Monday February 24, 2014

**Work all 4 problems.**

**Problem 1.** In class we showed

$$\begin{aligned} E[n(t)n(t+\tau)] &= \phi_{xx}(\tau)\cos 2\pi f_c t \cos 2\pi f_c(t+\tau) \\ &\quad +\phi_{yy}(\tau)\sin 2\pi f_c t \sin 2\pi f_c(t+\tau) \\ &\quad -\phi_{xy}(\tau)\sin 2\pi f_c t \cos 2\pi f_c(t+\tau) \\ &\quad -\phi_{yx}(\tau)\cos 2\pi f_c t \sin 2\pi f_c(t+\tau) \end{aligned}$$

and concluded

$$\phi_{nn}(\tau) = \phi_{xx}(\tau)\cos 2\pi f_c\tau - \phi_{yx}(\tau)\sin 2\pi f_c\tau.$$

Show how the last result is obtained from the prior expression.

**Problem 2.** Show that

$$\Phi_{nn}(f) = \int_{-\infty}^{\infty}\left[\mathrm{Re}(\phi_{zz}(\tau)e^{j2\pi f_c\tau})\right]e^{-j2\pi f\tau}d\tau$$

becomes

$$\Phi_{nn}(f) = \frac{1}{2}\left[\Phi_{zz}(f-f_c) + \Phi_{zz}(-f-f_c)\right].$$

**Problem 3.** We stated in class that the equivalent lowpass noise $z(t)$ has a power spectral density

$$\Phi_{zz}(f) = \begin{cases} N_0, & |f| \le B/2 \\ 0, & |f| > B/2. \end{cases}$$

Show that the corresponding autocorrelation function is

$$\phi_{zz}(\tau) = N_0\frac{\sin \pi B\tau}{\pi\tau}.$$

**Problem 4.** Hilbert transforms are sometimes encountered in the analysis of narrowband systems. Given a real-valued function $x(t)$, its Hilbert transform, denoted $\hat{x}(t)$, is defined by

$$\hat{x}(t) = x(t) * h(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau$$

where

$$h(t) = \frac{1}{\pi t}$$

with corresponding frequency response

$$H(f) = \begin{cases} -j, & f > 0 \\ 0, & f = 0 \\ j, & f < 0. \end{cases}$$

The integral above is to be interpreted as the principal value of the integral, that is, for a function $g(t)$

$$\int_{-\infty}^{\infty} g(t) dt$$

means

$$\lim_{\epsilon \to 0} \left[ \int_{-\infty}^{-\epsilon} g(t) dt + \int_{\epsilon}^{\infty} g(t) dt \right].$$

a. Show $\hat{\hat{x}}(t) = -x(t)$, that is, the Hilbert transform of the Hilbert transform of $x(t)$ equals $-x(t)$.

b. Show the Hilbert transform of $x(t) = \cos(\omega t + \phi)$ is $\hat{x}(t) = \sin(\omega t + \phi)$.

c. Using [a] and [b] show that the Hilbert transform of $y(t) = \sin(\omega t + \phi)$ is $\hat{y}(t) = -\cos(\omega t + \phi)$.