EE 564

Homework 3 Solutions

Problem 1. In class we gave two different demodulators for BPSK signals. Draw the circuits corresponding to these demodulators and show that they are equivalent by writing down a mathematical description of the signal as it pass thru the circuits.

Solution: See below for circuit. We find

$$
\begin{aligned}
r_{1a} &= \frac{A}{2}\cos\theta + \frac{A}{2}\cos(4\pi f_c t + 2\theta) \\
r_{1b} &= \frac{AT}{2}\cos\theta
\end{aligned}
$$

where we assume an integer number of sinusoidal cycles per integration period T. Also,

$$
\begin{aligned}
r_{2a} &= \frac{A}{2}\cos\theta + \frac{A}{2}\cos(4\pi f_c t + 2\theta) \\
r_{2b} &= \frac{AT}{2}\cos\theta \\
r_{3a} &= \frac{A}{2}\cos(\theta - \pi) + \frac{A}{2}\cos(4\pi f_c t + \theta + \pi) \\
r_{3b} &= \frac{AT}{2}\cos(\theta - \pi).
\end{aligned}
$$

We see that if $\theta = 0$ then $r_{2b} = r_{1b}$ and if $\theta = \pi$ then $r_{3b} = -r_{1b}$ so the two circuits make equivalent decisions.

Problem 2. In the first BPSK demodulator that we showed in class we had a single integrator in the circuit that integrated the received signal for T seconds (the period). In the presence of additive Gaussian noise the integrated noise component may be regarded as a normal (Gaussian) random variable with mean 0 and variance σ^2. Write down the density functions for the signal at the output of the integrator and draw them on a graph. Your graph should have two densities, for each value of the phase (0 or π). You may assume for your graph that the amplitude of the signal is $A = 2$ and

$\sigma^2 = 1$.

Solution: See below.

Problem 3. In class we let the signal $s(t)$ be real-valued:

$$s(t) = a(t)\cos[2\pi f_c t + \theta(t)] \tag{1}$$

$$a(t) = \textit{amplitude (or envelope) of}\, s(t)$$
$$\theta(t) = \textit{phase of}\, s(t)$$
$$f_c = \textit{carrier frequency of}\, s(t)$$

If bandwidth is much smaller than f_c, we have a bandpass system.

$$s(t) = a(t)\cos(\theta(t))\cos(2\pi f_c t) - a(t)\sin(\theta(t))\sin(2\pi f_c t)$$

$$= x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) \tag{2}$$
$$x(t) = a(t)\cos(\theta(t)) \quad \longrightarrow \quad \textit{in phase component}$$
$$y(t) = a(t)\sin(\theta(t)) \quad \longrightarrow \quad \textit{quadrature component}$$

$x(t)$ and $y(t)$ are low-pass signals, since their frequency component is concentrated around $f = 0$.

Let

$$u(t) = a(t)e^{i\theta(t)}$$
$$= x(t) + iy(t)$$

Then,

$$s(t) = Re\{u(t)e^{i2\pi f_c t}\} \tag{3}$$

We derived that the energy in $s(t)$ is

$$\xi = \int_{-\infty}^{\infty} s^2(t)dt$$

$$= \int_{-\infty}^{\infty} \{Re[u(t)e^{i2\pi f_c t}]\}^2 dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2 dt + \underbrace{\frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2 \cos[4\pi f_c t + 2\theta(t)]dt}_{\textit{small compared to the } 1^{st} \textit{ integral}}$$

So,

$$\xi \approx \frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2 dt$$

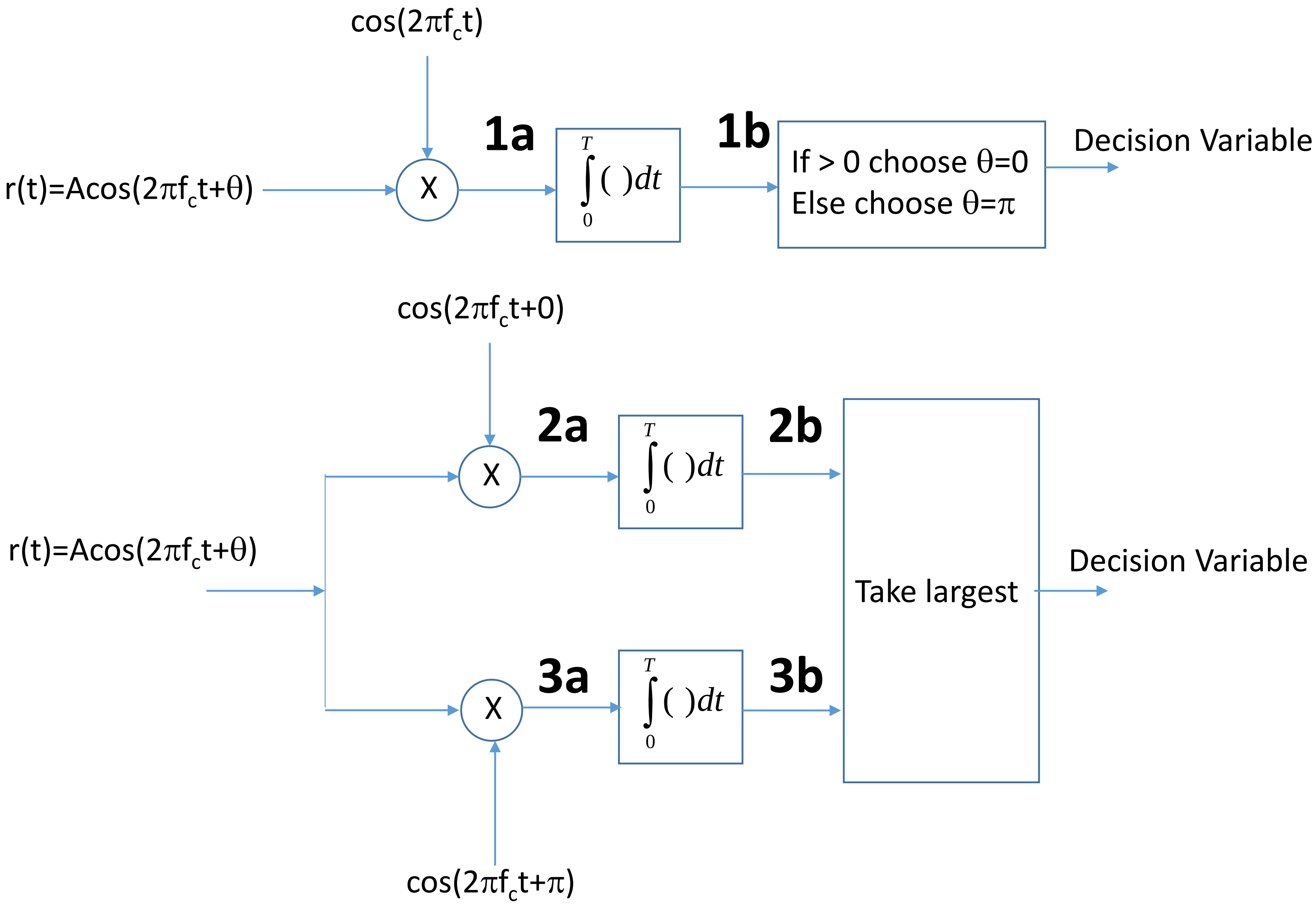
where, $|u(t)| = a(t)$, the envelope.

Justify the last step we took where we ignored the second integral.
Hint: You may assume that $u(t)$ is an energy signal and then show that the second integral goes to 0 as $f_c \to \infty$.

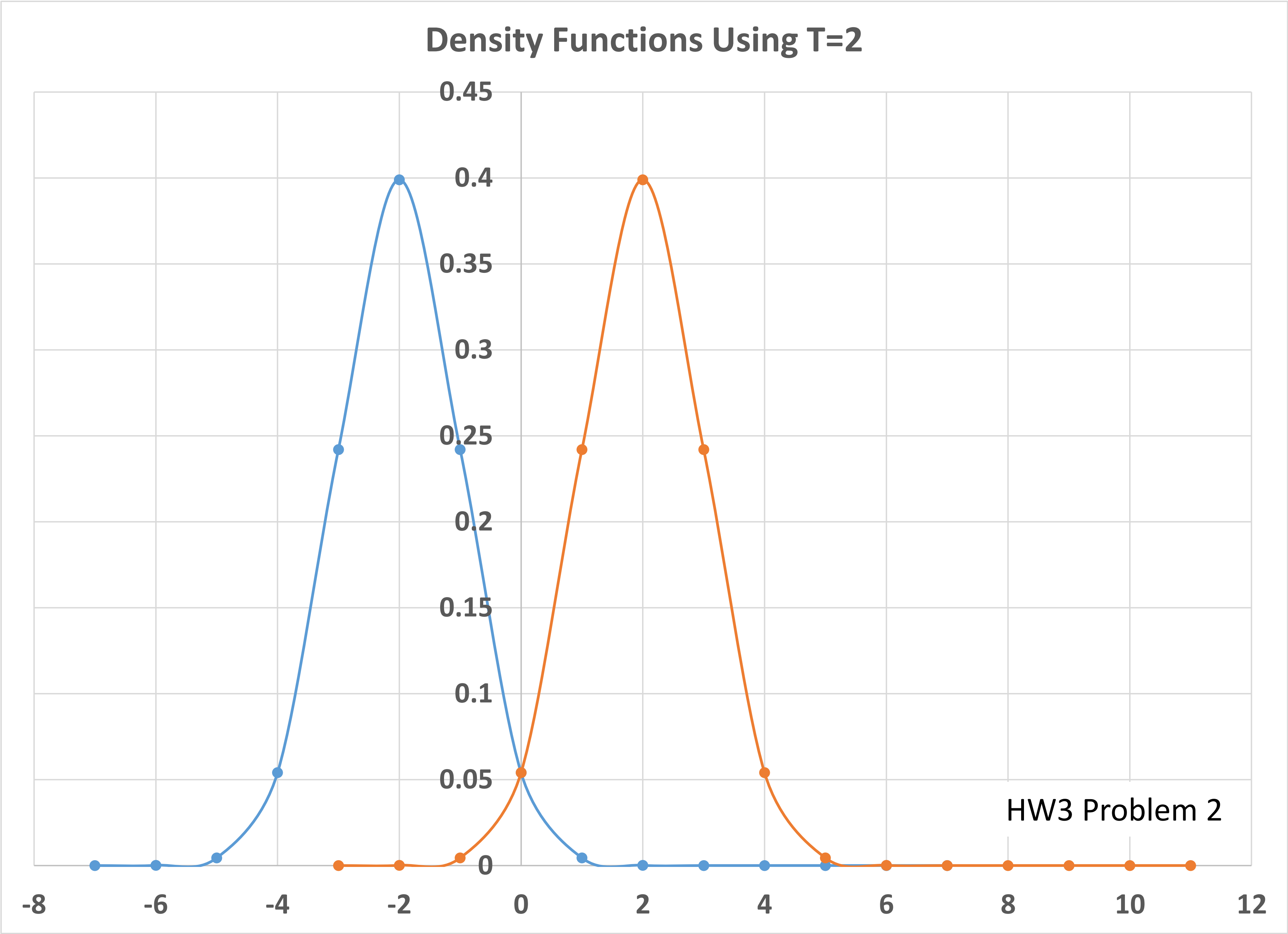
Solution: Since $u(t)$ is an energy signal it is finite almost everywhere and since it is a signal it is bounded, say $|u(t)| \leq M$, some finite M. Hence,

$$\begin{aligned}
\frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2 \cos[4\pi f_c t + 2\theta(t)]dt &\leq \frac{M^2}{2}\int_{-\infty}^{\infty} \cos[4\pi f_c t + 2\theta(t)]dt \\
&= \frac{M^2}{2}\lim_{T\to\infty}\int_{-T}^{T} \cos[4\pi f_c t + 2\theta(t)]dt \\
&= \frac{M^2}{2}\lim_{T\to\infty}\frac{1}{4\pi f_c}\sin[4\pi f_c t + 2\theta(t)]\bigg|_{-T}^{T} \\
&\leq \frac{M^2}{4\pi f_c}
\end{aligned}$$

and this last expression clearly goes to zero as $f_c \to \infty$.



HW3 Problem 1



Density Functions Using T=2
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-8
-6
-4
-2
0
2
4
6
8
10
12
HW3 Problem 2