EE 564

Homework 3

Due Monday February 10, 2014

Work all 3 problems.

Problem 1. In class we gave two different demodulators for BPSK signals. Draw the circuits corresponding to these demodulators and show that they are equivalent by writing down a mathematical description of the signal as it pass thru the circuits.

Problem 2. In the first BPSK demodulator that we showed in class we had a single integrator in the circuit that integrated the received signal for T seconds (the period). In the presence of additive Gaussian noise the integrated noise component may be regarded as a normal (Gaussian) random variable with mean 0 and variance σ^2. Write down the density functions for the signal at the output of the integrator and draw them on a graph. Your graph should have two densities, for each value of the phase (0 or π). You may assume for your graph that the amplitude of the signal is $A = 2$ and $\sigma^2 = 1$.

Problem 3. In class we let the signal $s(t)$ be real-valued:

$$s(t) = a(t)\cos[2\pi f_c t + \theta(t)] \tag{1}$$

$$a(t) = \textit{amplitude (or envelope) of}\, s(t)$$

$$\theta(t) = \textit{phase of}\, s(t)$$

$$f_c = \textit{carrier frequency of}\, s(t)$$

If bandwidth is much smaller than f_c, we have a bandpass system.

$$s(t) = a(t)\cos(\theta(t))\cos(2\pi f_c t) - a(t)\sin(\theta(t))\sin(2\pi f_c t)$$

$$= x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) \tag{2}$$

$$x(t) = a(t)\cos(\theta(t)) \quad \longrightarrow \quad \textit{in phase component}$$

$$y(t) = a(t)\sin(\theta(t)) \quad \longrightarrow \quad \textit{quadrature component}$$

$x(t)$ and $y(t)$ are low-pass signals, since their frequency component is concentrated around $f = 0$.
Let

$$\begin{aligned} u(t) &= a(t)e^{i\theta(t)} \\ &= x(t) + iy(t) \end{aligned}$$

Then,

$$s(t) = Re\{u(t)e^{i2\pi f_c t}\} \tag{3}$$

We derived that the energy in $s(t)$ is

$$\begin{aligned} \xi &= \int_{-\infty}^{\infty} s^2(t)dt \\ &= \int_{-\infty}^{\infty} \{Re[u(t)e^{i2\pi f_c t}]\}^2 dt \\ &= \frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2 dt + \underbrace{\frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2 \cos[4\pi f_c t + 2\theta(t)]dt}_{small\ compared\ to\ the\ 1^{st}\ integral} \end{aligned}$$

So,

$$\xi \approx \frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2 dt$$

where, $|u(t)| = a(t)$, the envelope.

Justify the last step we took where we ignored the second integral.
Hint: You may assume that $u(t)$ is an energy signal and then show that the second integral goes to 0 as $f_c \to \infty$.