EE 564

Homework 2 Solutions

Problem 1. Draw a PCM waveform using NRZ-L logic that encodes the binary data 1 0 1 1 0. Here T is 1 μsec.

Solution: See below.

Problem 2. Draw a Manchester encoded waveform using Bi-ϕ-L logic that encodes the binary data 1 0 1 1 0. Here T is 1 μsec.

Solution: See below.

Problem 3. Suppose you use PCM NRZ-L signaling direct to a receiver. By sampling the waveform in the middle of each pulse the receiver can try and recover the transmitted information with $+V = 1$ and $-V = 0$ ideally. Suppose the waveform is corrupted with additive noise. Now when we sample it we get a random variable so when a 1 is sent the sample becomes $V + N$ and when a 0 was sent the sample becomes $-V + N$. In this case if our sample is greater than or equal to 0 we will decide a 1 bit was sent and if our sample is less than 0 we decide upon a 0 bit. If a 1 is sent and our sample due to noise is less than 0 we would make an error. Similarly, if a 0 is sent we make an error if our sample is greater than or equal to 0. Suppose N is a normal (or Gaussian) random variable with mean 0 and variance 0.25 and $V = 1$. Compute the probability of making a decision error.

Solution:

$$P(error) = P(error|0\ sent)P(0\ sent) + P(error|1\ sent)P(1\ sent).$$

$$\begin{aligned} P(error|0\ sent) &= P(-V + N \geq 0) \\ &= P(N \geq V) \\ &= P(N \geq 1) \\ &= \int_1^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2(0.25)}\right) \end{aligned}$$

$$= \quad 0.0228$$

Similarly, $P(error|1\ sent) = 0.0228$. Hence,

$$\begin{aligned} P(error) &= 0.0228 \cdot P(0\ sent) + 0.0228 \cdot P(1\ sent) \\ &= 0.0228 \cdot (P(0\ sent) + P(1\ sent)) \\ &= 0.0228 \cdot (1) \\ &= 0.0228. \end{aligned}$$

Problem 4. A BPSK signal is given by

$$s(t) = \begin{cases} A\cos(2\pi f t + \phi), & 0 \le t \le T \\ 0, & \text{elsewhere,} \end{cases}$$

where $\phi = 0$ if a 1 is sent and $\phi = \pi$ if a 0 is sent and $A > 0$. One way to recover the information bit as discussed in class is to multiply $s(t)$ by $\cos(2\pi f t)$ and integrate the result from 0 to T and then decide upon 1 if the output is positive and decide upon 0 if the output is negative. Let y represent the output of the integration. Suppose $T = 1$ sec, $f = 4$ Hz, $\phi = 0$ and $A = 1$.

a. Compute y using the algorithm logic provided above.

   Solution:

$$\begin{aligned} y &= \int_0^1 \cos(2\pi 4t)\cos(2\pi 4t)dt \\ &= \int_0^1 \frac{1}{2}dt + \int_0^1 \frac{1}{2}\cos(16\pi t)dt \\ &= \frac{1}{2}. \end{aligned}$$

   We thus conclude a 1 was sent.

b. Now suppose T is unknown to the receiver so the receiver uses T_{est} for its integration time. Compute $y = y(T_{est})$ using $T_{est} = 0.1, 0.2, \ldots, 1.5$ and plot your results on a graph of y vs. T_{est}.

   Solution:If the next bit equals 1 we get

$$\begin{aligned} y &= \int_0^{T_{est}} \cos(2\pi 4t)\cos(2\pi 4t)dt \\ &= \int_0^{T_{est}} \frac{1}{2}dt + \int_0^{T_{est}} \frac{1}{2}\cos(16\pi t)dt \\ &= \frac{T_{est}}{2} + \frac{1}{16\pi}\sin(16\pi T_{est}). \end{aligned}$$

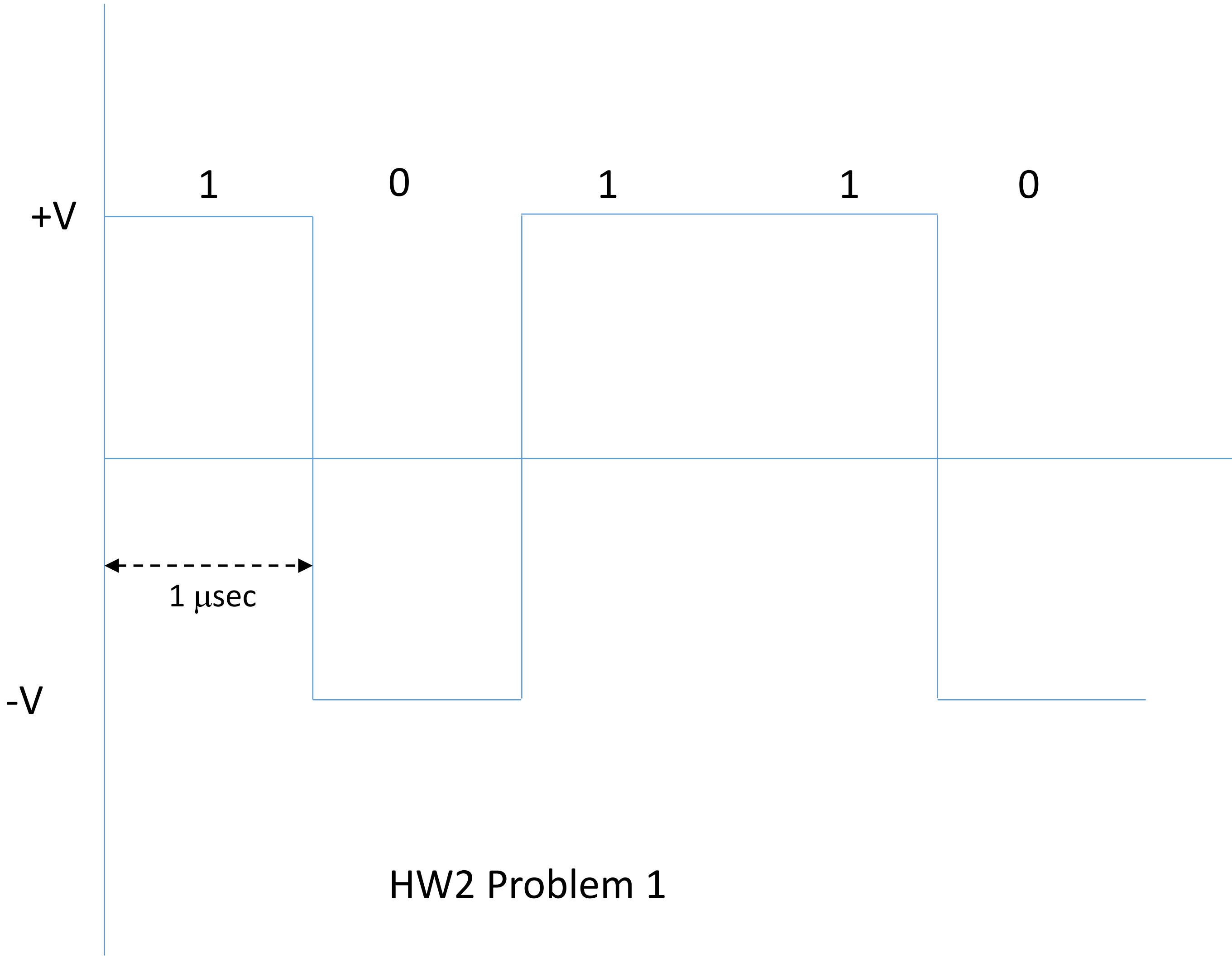
If the next bit equals 0 we get for $T_{est} \leq 1$

$$\begin{aligned} y &= \int_0^{T_{est}} \cos(2\pi 4t)\cos(2\pi 4t)dt \\ &= \frac{T_{est}}{2} + \frac{2}{16\pi}\sin(16\pi T_{est}) \end{aligned}$$

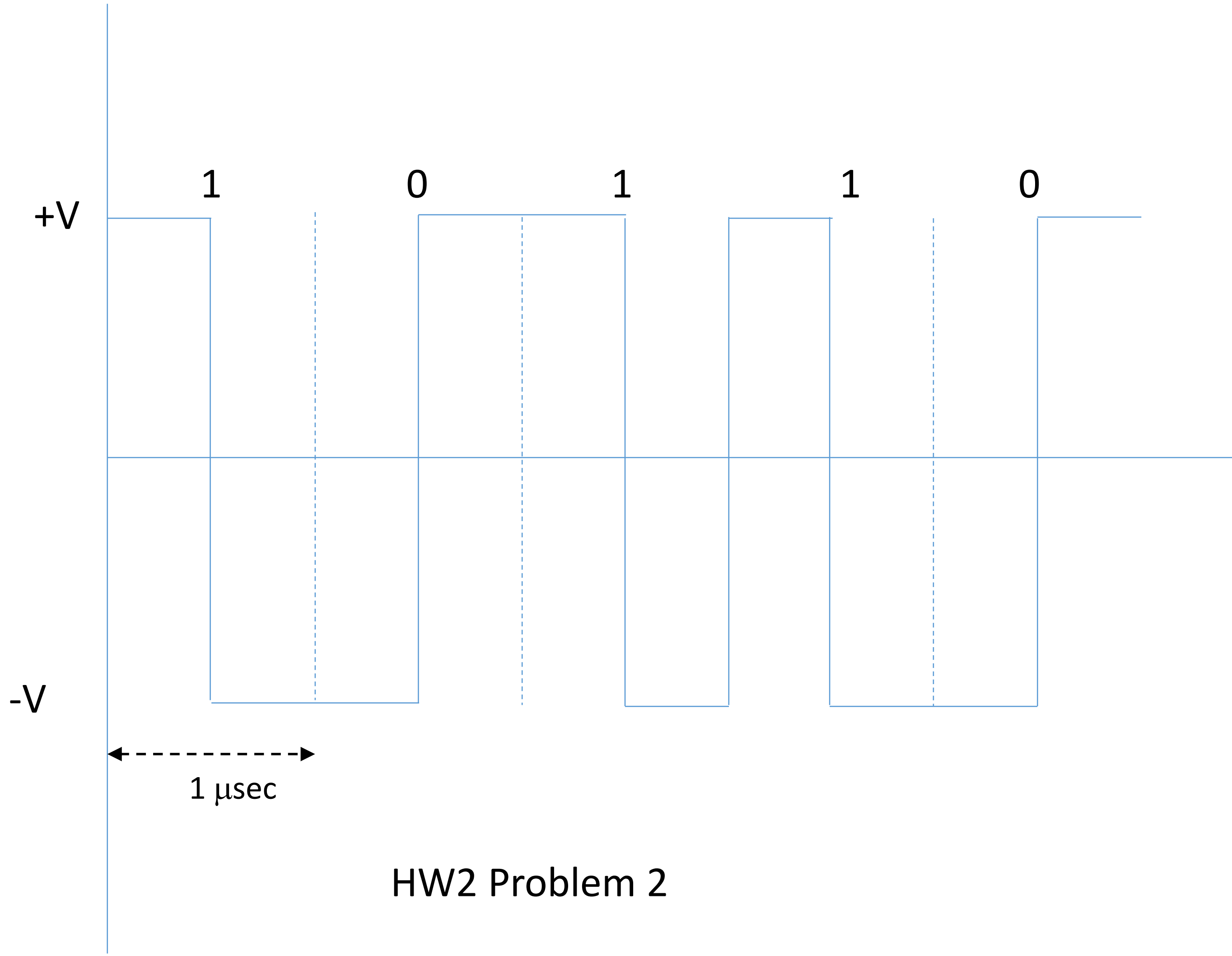
and for $T_{est} > 1$

$$\begin{aligned} y &= \int_0^{1} \cos(2\pi 4t)\cos(2\pi 4t)dt - \int_1^{T_{est}} \cos(2\pi 4t)\cos(2\pi 4t)dt \\ &= \frac{2 - T_{est}}{2} - \frac{1}{16\pi}\sin(16\pi T_{est}). \end{aligned}$$

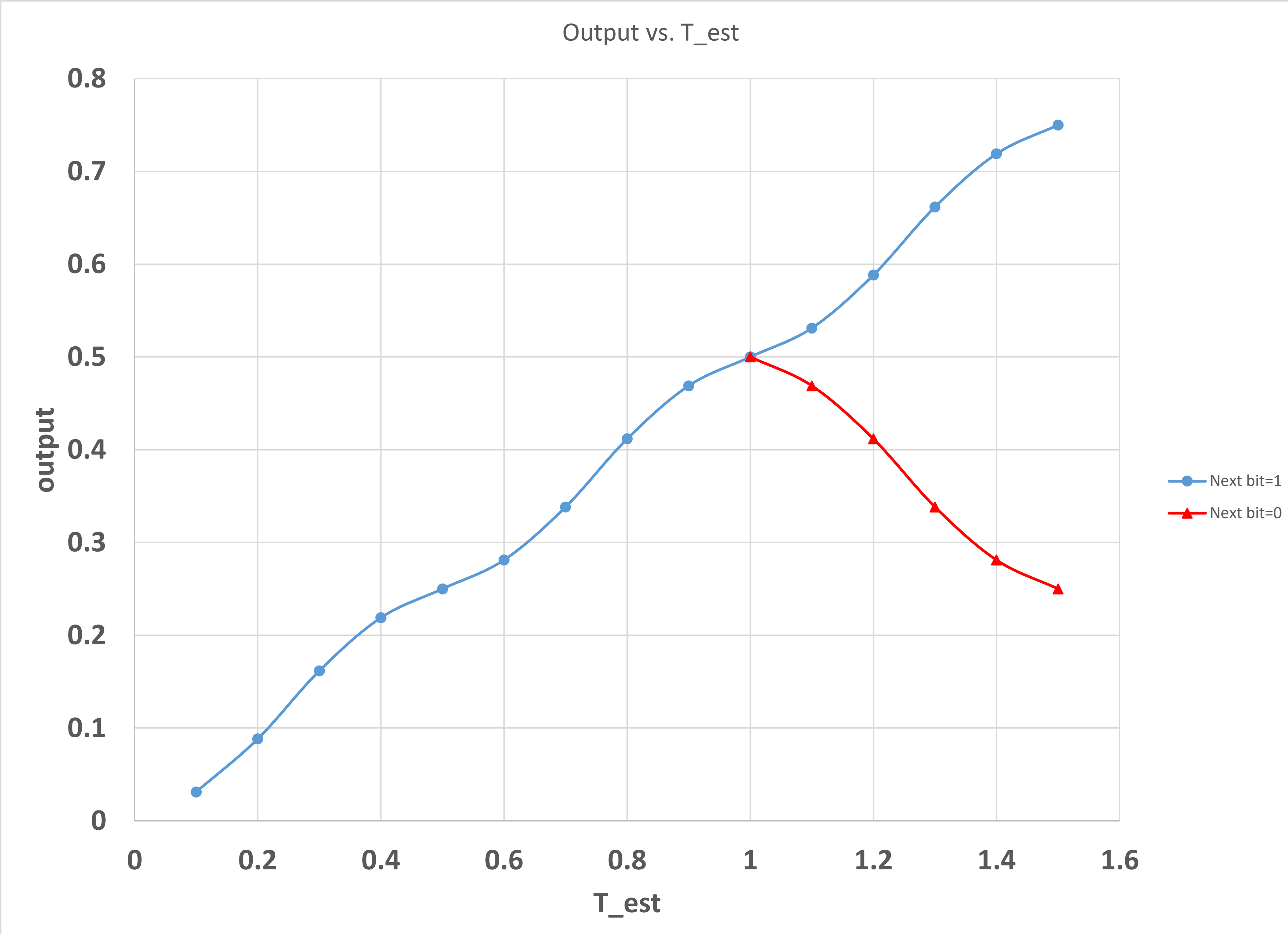
See graph below.



HW2 Problem 1



HW2 Problem 2



HW2 Problem 4