

EE 564

Homework 1

Due Monday January 27, 2014

Work all 8 problems.

Problem 1. The continuous random variable X has *pdf*

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

- a. Find $P(X \leq 2)$.
- b. Find $P(X \leq 2|X > 1)$.

Problem 2. Suppose the normal random variable X has mean 1 and variance 4. Let $Y = X^2 + 1$. Find the mean and variance of Y .

Problem 3. Suppose $X \sim N(2, 3)$, i.e., X is normally distributed with mean 2 and variance 3. Find the value of b that minimizes $E[(X - b)^2]$.

Problem 4. Suppose $X \sim N(2, 3)$. Find $E[X^4]$.

Problem 5. Suppose the random variable X has mean 0 and variance 4. Let $Y = 2X + 1$. Find the correlation coefficient, r_{XY} .

Problem 6. Suppose $X \sim N(1, 2)$, and $Y \sim N(0, 6)$, and X is independent of Y . Let $Z = X + Y$. Find the variance of Z .

Problem 7. The two-dimensional continuous random variable (X, Y) has joint *pdf*

$$f_{XY}(x, y) = \begin{cases} x + y, & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a. Compute $f_{Y|X}(y|x)$.
- b. Find $E[Y|X = x]$.

Problem 8. Suppose we toss a fair coin 50 times. Let X denote the number of heads obtained.

- a. Find $P(22 < X \leq 28)$ using the binomial distribution.
- b. Find $P(22 < X \leq 28)$ using the normal approximation. Write your answer using

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2} du.$$