EE 564

Final Exam

Due Monday, May 12, 2014 at 4:30 p.m.

Work all 6 problems.

Problem 1. [15 pts]. Suppose that BPSK modulation is used for transmitting information over an AWGN channel with a power spectral density of

$$\frac{1}{2}N_0 = 10^{-10}\ W/Hz.$$

The transmitted signal energy is

$$E_b = \frac{1}{2}A^2T.$$

where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve an error probability of 10^{-5} when the data rate is

a. 100 kbps.

b. 1 Mbps.

c. 10 Mbps.

Problem 2. [16 pts]. Consider a signal detector with an input

$$r = \pm A + n$$

where $+A$ and $-A$ occur with equal probability and the noise variance n is characterized by the Laplacian density

$$p(n) = \frac{1}{\sqrt{2}\sigma}e^{-|n|\sqrt{2}/\sigma}.$$

a. Determine the probability of error as a function of A and σ.

b. Find the SNR ($= A^2/\sigma^2$) required to achieve an error probability of 10^{-5}.

c. Repeat part (a) for the Gaussian density

$$p(n) = \frac{1}{\sqrt{2\pi}\sigma} e^{-n^2/2\sigma^2}.$$

d. Repeat part (b) for the above Gaussian density and compare the results you got using the Laplacian and Gaussian densities.

Problem 3. [21 pts]. A certain communication system transmits information using one of the following three waveforms

$$s(t) = \begin{cases} s_0(t) = 0, \\ s_1(t) = A, \\ s_2(t) = -A, \end{cases}$$

every T seconds. The received signal is

$$r_L(t) = s(t) + z(t)$$

where $z(t)$ is white Gaussian noise with $E[z(t)] = 0$ and $E[z(t)z^*(\tau)] = 2N_0\delta(t-\tau)$. The optimum receiver computes the correlation metric

$$U = \mathrm{Re}\left[\int_0^T r_L(t)s_1^*(t)dt\right]$$

and compares U to a threshold γ. If $U > \gamma$ the decision is made that $s_1(t)$ was sent. If $U < -\gamma$ the decision is $s_2(t)$. Otherwise, it is decided that $s_0(t)$ was sent.

a. If we assume that $s_1(t)$ was sent then show that the mean of U is $2E$ where E is the energy in $s_1(t)$ defined by (for this lowpass signal)

$$E = \frac{1}{2}\left[\int_0^T s_1(t)s_1^*(t)dt\right].$$

b. Show that the variance of U is $2EN_0$ regardless of the waveform sent.

c. Assuming that the three signals are equally likely to be sent determine the value of γ that minimizes the probability of decision error and determine what the average probability of error is for this ternary system.

Problem 4. [20 pts]. Suppose we have a QPSK constellation where each point is at a radius of r_1 from the origin. The performance of this system depends on the distance between the constellation points and is mostly determined by the distance between adjacent points, d. It is a simple application of the Pythagorean theorem to show that $r_1 = d/\sqrt{2}$ (verify this). With this system we transmit 2 bits per constellation symbol. Suppose now we consider an 8PSK constellation where we transmit 3 bits per symbol. If we used the same radius for the points as was used for the QPSK system the points would be closer to each other in distance since we have 8 points on the ring instead of 4. Thus, the probability of a symbol decision error at the receiver is higher for this design. Therefore, if we wish to maintain the same probability of symbol error for the 8PSK system that we had for the QPSK system we have to increase the radius of the ring to some value r_2 so that the distance between adjacent constellation points for the 8PSK system is also d.

a. Find the required value of the radius r_2 in terms of d.

b. How much more power is required for the 8PSK system with a radius of r_2 to achieve the same performance as the QPSK system.

Problem 5. [12 pts]. Suppose that the loop filter for the PLL as given in class has the transfer function

$$G(s) = \frac{1}{s + \sqrt{2}}.$$

a. Determine the closed loop transfer function $H(s)$ and indicate if the loop is stable.

b. Determine the damping factor and the natural frequency of the loop.

Problem 6. [16 pts]. A discrete memoryless source (DMS) has an alphabet of 8 letters, $x_i,\ i = 1, 2, \ldots, 8$ with corresponding probabilities

$$0.25,\ 0.20,\ 0.18,\ 0.10,\ 0.09,\ 0.08,\ 0.05,\ 0.05$$

a. Use the Huffman coding procedure to determine a binary code for this data.

b. Determine the average number $\bar{R}$ of binary digits per source letter.

c. Determine the entropy of the source and compare it to $\bar{R}$.