EE 567

Homework 7

Due Tuesday, October 15, 2019

Work all 3 problems.

Problem 1. In class we had that the gain for a parabolic antenna is

$$g = \left(\frac{\pi}{3}\right)^2 \left(\frac{d \cdot f_c}{10^8}\right)^2$$

and the half-power beamwidth is

$$\phi_b = 3.06 \left(\frac{10^8}{d \cdot f_c}\right) \quad \text{radians.}$$

The actual gain of the antenna will be reduced if there is any pointing error ϕ_e and the resulting equation is

$$g(\phi_e) = g \left[\frac{2J_1(\pi d \phi_e/\lambda)}{(\pi d \phi_e/\lambda)}\right]^2$$

where λ is the wavelength. Using the Bessel function approximation for small argument

$$J_1(x) \approx \frac{x}{2}\left(1 - \frac{x^2}{8}\right) \approx \frac{x}{2} e^{-x^2/8}$$

we obtain

$$g(\phi_e) \approx g e^{-2.6(\phi_e/\phi_b)^2}.$$

Plot the antenna gain (in dB) versus d/λ for $\phi_e = 0,\ 0.01,\ 0.05,\ 0.1,\ 0.15$ degrees. You should plot each of these on the same graph. Vary d/λ from 10 to 3000. You should be able to deduce from your results that pointing errors prevent the use of very narrow beams.

Problem 2. A signal of the form

$$s_1(t) = A\cos(2\pi f_c t + \theta)$$

is transmitted to a receiver. The signal waveform is T seconds long. A multipath version of the signal, $s_2(t)$, delayed by τ seconds also arrives at the receiver (you can think of this multipath signal as being just like $s_1(t)$ except it is delayed and the amplitude is possibly scaled). The cross-correlation of the two real signals is defined by

$$\gamma_{12} = \frac{1}{T}\int_0^T s_1(t)s_2(t)dt.$$

Determine the smallest value of τ that can be tolerated to ensure that the cross-correlation of the direct path and multipath signal is less than 5% of the direct signal power. Assume that $f_c T \gg 1$ and the multipath signal has an amplitude that is 70% that of the direct signal when received.

Problem 3. Use Matlab or some other tool. Suppose we receive the analog signal

$$r_a(t) = A\cos(2\pi 200t + \theta)$$

and sample it at 2000 Hz to get the digital signal

$$r(n) = A\cos(0.2\pi n + \theta).$$

Suppose now we quantize the digital signal to get

$$r_q(n) = \text{Round}\left[A\cos(0.20\pi n + \theta)\right]$$

where 'Round' means the samples are rounded to the nearest integer. The amplitude A is a constant but we do not know its value. Furthermore, we do not know that the phase is $\theta = \pi/4$. We can follow the steps below to estimate the value of A. [For purposes of calculation let A actually have the value 10.]

S1. Multiply $r_q(n)$ by $x_1(n)$ and $x_2(n)$, where $x_1(n) = \cos(0.2\pi n)$ and $x_2(n) = \sin(0.2\pi n)$. Call the results $y_1(n)$ and $y_2(n)$, respectively.

S2. Simply add up the values of $y_1(n)$ and $y_2(n)$ for $n = 0, 1, 2, \ldots N-1$ (some N) and take the average of each (divide by N) and then multiply the averages by 2. Call the results z_1 and z_2, respectively.

S3. Compute $\sqrt{z_1^2 + z_2^2}$. This is the estimate of A.

a. Follow the 3 steps above and estimate A using $N = 5$.

b. Repeat (a) for $N = 10$.

c. What values of N makes your estimate of A exact if the input samples were not quantized.

d. Based on your answers to parts (a) and (b) what would your estimate for A be if N is 1000. Explain why the estimate is not becoming exact even for very large N.

e. If we change the sampling frequency to $2000 \times \pi/3$ Hz will our estimate for A become exact for large N? If so, explain why and also show that the estimate becomes exact using Matlab or another tool.