EE 567

Homework 6

Due Tuesday, October 8, 2019

Work all 4 problems.

Problem 1. Let $X(t) = X(u,t)$ denote a random process described by

$$X(t) = \begin{cases} e^{-ut}, & t \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

where, u is a realization of a uniform (0,1) random variable. Define $Y(t) = Y(u,t)$ as follows:

$$Y(t) = \begin{cases} 1, & X(t) \geq e^{-2} \\ 0, & \text{elsewhere.} \end{cases}$$

Compute the correlation $R_Y(t_1, t_2)$.

Problem 2. Let $X_k,\ k = 1, 2, \ldots, n$ be a sequence of independent and identically distributed random variables each with mean μ and variance σ^2. Let

$$S^2 = \frac{1}{n-1} \sum_{k=1}^{n} \left(X_k - \bar{X} \right)^2 .$$

Show that $E\left[S^2\right] = \sigma^2$ (do not assume that X_k is Gaussian).

Hint: $X_k - \bar{X} = X_k - \mu + \mu - \bar{X}$.

Problem 3. Suppose $X_1, X_2, \ldots$ are each independent and normally distributed with mean zero and variance one (standard normal). Define $Y_1 = X_1$ and for $n = 2, 3, 4, \ldots$ let

$$Y_n = \alpha X_n + (1-\alpha) Y_{n-1}, \quad 0 < \alpha < 1.$$

a. Compute $E[Y_n Y_m]$ as a closed form function of n, m and α for $n \geq m$.

b. Evaluate your expression for $E[Y_n Y_m]$ using $n = 10$, $m = 5$ and $\alpha = 0.2$.

Problem 4. Consider a real Gaussian random sequence $x(n)$, n an integer, with

$$E[x(n)] = 0, \quad E\left[x(n)^2\right] = 1, \quad E[x(n)x(m)] = \rho^{|n-m|}$$

where $0 < \rho < 1$. Let

$$y(n) = 2x(n) + 2.$$

a. Is $x(n)$ wide sense stationary?

b. Find the covariance of $y(n)$ and state whether or not it is wide sense stationary.

Note: WSS has the same meaning here using the index n as in the lecture notes where we used t but now the "time" is discrete.